King Saud University
College of Engineering in Al-kharj

Solution

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Math 105
Level :2
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First Exam
Semester II (1428/429)

Date: 03-22-1429 **Max. MARKS: 15**

Time : 90min **Number of Pages : 05**

NAME : _____

REG.NO : _____

GROUP :_____

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Exercise	Marks
1	
2	
3	
4	
5	
Total	

Exercise1 (3 degrees):

1- $f(x)=\sqrt{1-x}$	2- $f(x)=\sqrt{\lvert x\rvert-2}$
$1-x\geq 0$ $-x\geq -1$ $x\leq 1 \quad \Rightarrow \quad D=(-\infty,1]$	$\lvert x\rvert-2\geq 0$ $\lvert x\rvert\geq 2$ $x\geq 2 \quad or \quad x\leq -2$ $\Rightarrow D=[2,+\infty)\cup(-\infty,-2]$
3- $f(x)=\dfrac{x-1}{x^2+1}$	4- $f(x)=\sqrt{x}-\sqrt{6-x}$
$D=R \quad because \quad x^2+1\succ 0$	$x\geq 0 \quad and \quad 6-x\geq 0$ $x\geq 0 \quad and \quad -x\geq 6$ $x\geq 0 \quad and \quad x\leq -6$ $\Rightarrow \quad D=[0,+\infty)\cap(-\infty,-6]$
5- $f(x)=\dfrac{1}{\sqrt{1-x}}$	6- $f(x)=\dfrac{x}{x^2+x-2}$
$1-x\succ 0$ $-x\succ -1$ $x\prec 1$ $\Rightarrow \quad D=(-\infty,1)$	$x^2+x-2\neq 0$ $(x-1)(x+2)\neq 0$ $\Rightarrow x-1\neq 0 \quad and \quad x+2\neq 0$ $\Rightarrow x\neq 1 \quad and \quad x\neq -2$ $\Rightarrow D=R-\{1,-2\}$

Exercise2 (4 degrees):

(a) $x+3\leq 0$	(b) $\dfrac{1}{x+2}\prec 0$
$x+3\leq 0 \quad \Rightarrow \quad x\leq -3$ *The solution set is* $(-\infty,-3]$	$\dfrac{1}{x+2}\prec 0 \quad \Rightarrow \quad x+2\prec 0 \quad \Rightarrow x\prec -2$ *The solution set is* $(-\infty,-2]$
(c) $x^2\leq 8$	(d) $x^2-1\geq 0$
$x^2\leq 8 \quad \Rightarrow \quad \sqrt{x^2}\leq\sqrt{8}$ $\Rightarrow \lvert x\rvert\leq\sqrt{8} \quad \Rightarrow \quad -\sqrt{8}\leq x\leq\sqrt{8}$ *The solution set is* $\left[-\sqrt{8},\sqrt{8}\right]$	$x^2-1\geq 0 \quad \Rightarrow \quad x^2\geq 1$ $\Rightarrow \sqrt{x^2}\geq 1 \Rightarrow \quad \lvert x\rvert\geq 1$ $\Rightarrow x\geq 1 \quad or \quad x\leq -1$ *The solution set is* $(-\infty,-1]\cup[1,+\infty)$

(e) $\lvert 3-x\rvert+3-x\geq 2$
If $x\in(-\infty,3]$, $\lvert x-3\rvert=x-3$ *so*, $3-x+3-x\geq 2\Rightarrow 6-2x\geq 2 \quad\Rightarrow\quad -2x\geq -4 \quad\Rightarrow\quad x\leq 2$ I*f* $x\in[3,+\infty)$, $\lvert 3-x\rvert=-3+x$ *so*, $-3+x=3-x\geq 2\Rightarrow 0\geq 2$ *impossible* *Then the solution set is* :$(-\infty,2]$

Exercise3 (2 degrees):

(i)

$$\begin{aligned} a<b &\Rightarrow a+b<b+b \\ &\Rightarrow a+b<2b \\ &\Rightarrow \frac{a+b}{2}<b \\ &\Rightarrow \frac{a}{2}+\frac{b}{2}<b \end{aligned}$$

(ii)

$$\begin{aligned} \lvert x+3\rvert<\frac{1}{2} &\Rightarrow -\frac{1}{2}<x+3<\frac{1}{2} \\ &\Rightarrow -2<4x+12<2 \qquad (\times 4) \\ &\Rightarrow -1<4x+13<3 \qquad (+1) \\ -3<-1 &\Rightarrow -3<4x+13<3 \\ &\Rightarrow \lvert 4x+13\rvert<3 \end{aligned}$$

Exercise4 (2 degrees):

- (fog)(x) = f(g(x))=f(2x) = 2x-1
- (gof)(x) = g(f(x)) =g(x-1) = 2(x-1) = 2x-2
- (fog)(1) = 2*1 - 1 = 2 - 1= 1
- (fog)(3x-1) = 2(3x-1) -1 = 6x -2-1 = 6x-3

Exercise5 (4 degrees):

- Coordinates of P_1 and P_2 (x-coordinate and y-coordinate).

P_1 (0,1) ; P_2(2,3)

- distance between P_1 and P_2.

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2-0)^2 + (3-1)^2} = \sqrt{8}$$

- The slope of the line.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{2-0} = \frac{2}{2} = 1$$

- y-intercept and x-intercept.

By using the graph: y − intercept is : $y = 1$

x − intercept is : $x = -1$

- The equation of the line.

the equation is : $y = mx + b$ *where b is* $y - \text{int}\,ercept \Rightarrow y = x + 1$