REAL NUMBERS (Review)

1 Classification of numbers

This classification of Numbers represents the most accepted elementary classification, and is useful in computing sense.

Class	Symbol	Description
Natural Number	$\mathbb{N}$	Natural numbers are defined as non-negative counting numbers: $\mathbb{N}$ = { 0, 1, 2, 3, 4, ... }. Some exclude 0 (zero) from the set: $\mathbb{N}$ * = $\mathbb{N}$ \{0} = { 1, 2, 3, 4, ... }.
Integer	$\mathbb{Z}$	Integers extend $\mathbb{N}$ by including the negative of counting numbers: $\mathbb{Z}$ = { ..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ... }.
Rational Number	$\mathbb{Q}$	A rational number is the ratio or quotient of an integer and another non-zero integer: $\mathbb{Q}$ = {n/m | n, m ∈ $\mathbb{Z}$, m ≠ 0 }. E.g.: -100, -20¼, -1.5, 0, 1, 1.5, 1½ 2¾, 1.75, &c
Irrational Number		Irrational numbers are numbers which cannot be represented as fractions. E.g.: √2, √3;, π, e.
1-1 Real Number	$\mathbb{R}$	Real numbers are all numbers on a number line. The set of $\mathbb{R}$ is the union of all rational numbers and all irrational numbers.

Table 1-1 : Classification of numbers

2- Coordinate Lines (Real number line)

We can arrange all the Real numbers on a number line. A number line is a horizontal line that has a positive direction, an origin and unit of measurement.

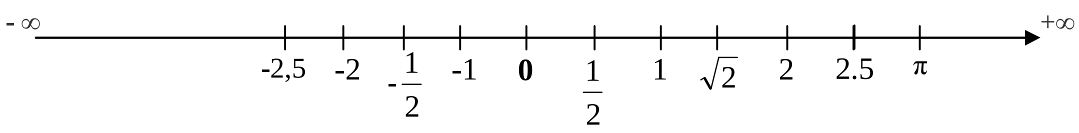


Figure 1.1: Real Line

Each number is greater than all the numbers to its left and less than all the numbers to its right. For example, 1 is greater than 0 and -2, but less than $\sqrt{2}$ and 3.

The location of π and $\sqrt{2}$, which are approximate, were obtained from the decimal approximations, π = 3.14 and $\sqrt{2}$ = 1.41.

3- Order properties

The real numbers can be ordered by size as follows :
If **b–a is positive**, then we say that **b is greater than a** or **a is less than b** and write **a <b**
If **b–a is negative**, then we say that **a is greater than b** or **b is less than a** and write **a >b**
The inequality **a ≤ b** is defined to mean that **a<b** or **a=b**.
Table1-2 lists various types of inequalities and their illustrations.

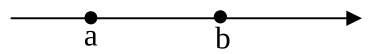
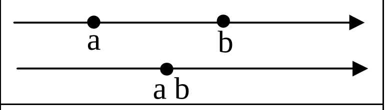
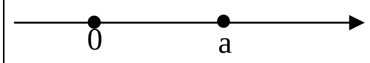
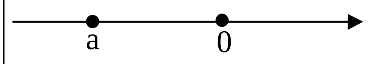
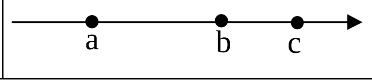
Inequality	Geometric interpretation	Illustration
a < b or b >a	a is to the left of b	
a ≤ b or b ≥ a	a is to the left of b or coincides with b	
0 < a or a > 0	a is to the right of the origin	
a < 0 or 0 > a	a is to the left of the origin	
a < b < c	a is to the left of b and b is to the left of c	

Table 1-2: various types of inequalities

Theorem. Let **a, b, c** and **d** be real numbers

(i) If a < b and b < c, then a < c
(ii) If a < b, then a + b < b + c and a - b < b - c
(iii) If a < b, then ac < bc when c is positive and ac > bc when c is negative
(iv) If a < b and c < d, then a + c < b+ d
(v) If a and b are both positive or both negative and a < b, then $\frac{1}{a} > \frac{1}{b}$

This properties remain true if < and > are replaced by ≤ and ≥.

Example :
If x < y, then -3x > -3y
If x < y, then 2x < 2y
If a > b, then a - 5 > b – 5
If 0 < x + 1 < y – 6, then $\frac{1}{x+1} > \frac{1}{y-6}$

4- Intervals :

Geometrically, an interval is a line segment on a coordinate line.

If a and b are real numbers such that $a < b$, then the **closed interval** from a to b is denoted by **[a, b]** and is defined by

$$[a, b] = \{x: a \leq x \leq b\}$$

and the **open interval** from a to b is denoted by **(a, b)** and is defined

$$(a, b) = \{x: a < x < b\}$$

where $\{x: a < x < b\}$ is read *"the set of all x such that a < x< b"*

The square brackets indicate that the endpoints are included in the interval and the parentheses indicate that they are not.

Table1.3 lists the various types of intervals. In that table, the geometric picture use solid dots to denote endpoints that are included in the interval and open dots to denote endpoints that are not.

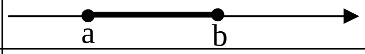
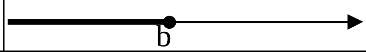
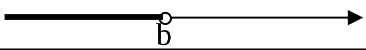
Interval notation	**Set notation**	**Geometric picture**	**Classification**
[a, b]	$\{x: a \leq x \leq b\}$		Finite; closed
(a, b)	$\{x: a < x < b\}$		Finite; open
[a, b)	$\{x: a \leq x < b\}$		Finite; half-open or half-closed
(a, b]	$\{x: a < x \leq b\}$		Finite; half-open or half-closed
$(-\infty, b]$	$\{x: x \leq b\}$		Infinite; closed
$(-\infty, b)$	$\{x: x < b\}$		Infinite; open
$[a, +\infty)$	$\{x: x \geq a\}$		Infinite; closed
$(a, +\infty)$	$\{x: x > a\}$		Infinite; open
$(-\infty, +\infty)$	{x: x is real number}		Infinite; open and closed

Table1.3: various types of intervals

5- Solving inequality :

A solution of inequality in an unknown x is a value for x that makes the inequality a true statement. For example, 1 is a solution of the inequality $x < 5$, but $x = 7$ is not.

Example 1 :

Solve $3 + 7x \le 10x + 9$

Solution. To solve inequality means to determine the set numbers x for which the inequality is true. This is called the solution set.

We shall use the operations of Theorem to isolate x on one side of the inequality.

First we subtract 3 from each side of the inequality:

$$3 + 7x - 3 \le 10x + 9 - 3 \qquad \Rightarrow \qquad 7x \le 10x + 6$$

Then we subtract 10x from both sides:

$$7x - 10x \le 10x + 6 - 10x \qquad \Rightarrow \qquad -3x \le 6$$

Now we divide both sides by -3:

$$x \ge -\frac{6}{3} = -2$$

So the solution set consists of all numbers greater than -2. In other words, the solution of the inequality is the interval $[-2,+\infty)$

Example 2:

Solve the inequalities $\qquad 4 \le 3x - 2 < 13$

Solution. Here the solution set consists of all values of x that satisfy both inequalities.

We see that the following inequalities are equivalent:

$$4 \le 3x - 2 < 13$$

$$6 \le 3x < 11 \qquad \text{(add 2)}$$

$$2 \le x < 5 \qquad \text{(divide by 3)}$$

Therefore, the solution set is $[2, 5)$.

Example 2:

Solve the inequality $\qquad x^2 - 5x + 6 \le 0$

Solution. First we factor the left side :

we know that the corresponding equation $(x - 2)(x - 3) = 0$ has two solutions 2 and 3. The numbers 2 and 3 divide the real line into Three intervals: $(-\infty, 2)$ $\quad (2, 3)$ $\quad (3, +\infty)$

On each of these intervals we determine the signs of the factors. For instance, x $\in$ (-∞, 2) $\Rightarrow$ x < 2 $\Rightarrow$ $\begin{cases} x-2 \prec 0 \\ x-3 \prec 0 \end{cases}$

Then we record these signs in the following chart:

Interval	x - 2	x - 3	(x - 2)(x – 3)
x < 2	-	-	+
2 < x <3	+	_	-
x > 3	+	+	+

The chart show that (x - 2)(x – 3) is negative when 2 < x <3. Thus, the solution of inequality $x^2 - 5x + 6 \leq 0$ is:

{x: 2 ≤ x ≤ 3} = [2, 3]

6-Absolute Value

The absolute value of a number a, denoted by $|a|$, is the distance from a to 0 On the real number line. Distances are always positive or 0, so we have

$$|a| \geq 0 \text{ for every number a}$$

In general, we have

$|a|$=a	if a ≥ 0
$|a|$=-a	if a < 0

For example

$|3| = 3$ $|-3| = 3$ $|\sqrt{2}-1| = \sqrt{2}-1$ $|3-\pi| = \pi - 3$

13 + | 2 - 3| + | 5×6| = 13 + | - 1| + | 30| = 13 + 1 + 30 = 44

34 - | 3×(- 4)| + | 4×2 - (- 5)| = 34 - | - 12| + | 13| = 34 - 12 + 13 = 35

6-1 Properties of absolute values

Suppose then **a** and **b** are any real number and **n** is an integer. Then

(i) $|ab| = |a||b|$ (ii) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $(b \neq 0)$) (iii) $|a^n| = |a|^n$

(iv) $|x| = a \quad if\ and\ only\ if\ x \pm a$

(v) $|x| < a \quad if\ and\ only\ if\ -a < x < a$

(vi) $|x| > a \quad if\ and\ only\ if\ x > a\ \ or\ \ x < -a$

6-2 Example :

- Express $|3x-2|$ without using the absolute value symbol.
- Solve $|6x-4| = 3$
- Solve $|3x+2| > 3$

Solution: