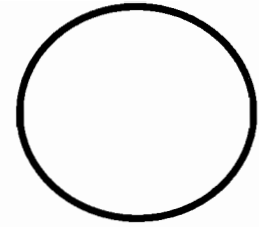


M - 105



MAX. MARKS: 100

TIME : 3 Hours

DEPARTMENT OF MATHEMATICS

KING SAUD UNIVERSITY

Final Examination Semester II (1426/1427)

Number of Pages:9.

NAME : _____

REG.NO: _____

GROUP NO: _____

NAME OF TEACHER: _____

Question No	Marks
1 - 20	
21,22	
23,24	
25,26	
27	
28,29	
Total Marks	

Mark {a,b,c,or d} for the correct answer in the space below for Q.1. - Q.20

Q.No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a,b,c,d																				

Marks : 2 x 20 = 40

Q.1. The solution set of the inequality $|3x - 4| < 5$ is

- (a) $(-\infty, -\frac{1}{3})$, (b) $(3, +\infty)$, (c) $(-\frac{1}{3}, 3)$, (d) None of these.

Q.2. The domain of the function $f(x) = \frac{x^2 + x + 1}{\sqrt{x^2 - 2x + 1}}$, is

- (a) $(-\infty, -1)$, (b) \mathbb{R} , (c) $\mathbb{R} - \{1\}$, (d) None of these.

Q.3. If $f(x) = \sin(x + g(x))$ and $f(0) = 0$ then $g(0)$ is equal to

- (a) 1, (b) 0, (c) $\frac{\pi}{2}$, (d) None of these.

Q.4. If $f(x) = \frac{x^2 - 5x + 6}{x - 3}$ is continuous at $x = 3$, then $f(3)$ is equal to

- (a) 1, (b) -1, (c) 2, (d) None of these.

Q.5. If $\lim_{x \rightarrow 2} \left[\frac{x^2}{x-2} - \frac{4}{x-2} \right]$, is equal to

- (a) 4, (b) -4, (c) 2, (d) None of these.

Q.6. If $f(x) = |x| \sin x$, then $f'(0)$ is

- (a) 0 (b) 1, (c) not defined, (d) None of these.

Q.7. The horizontal asymptote to the graph of $f(x) = \frac{3x^2 - x}{3 + x^2}$ is

- (a) $y = 3$, (b) $x = 1$, (c) $y = -1$, (d) $x = 3$.

Q.8. A function $f(x) = 1 + x^{\frac{2}{5}}$ has a cusp at the point

- (a) (0,0), (b) (1,0), (c) (0,1), (d) None of these.

Q.9. The liner approximation of the $f(x) = (1+x)^3$ near $a = 0$ is

- (a) $1+x$, (b) $1+3x$, (c) $(1+x)^2$, (d) none of these

Q.10. Rate of increase of area of a square when its width x increases at the rate 4 cm/sec, and its length is 5 cm is

- (a) $10 \text{ cm}^2/\text{sec}$, (b) $40 \text{ cm}^2/\text{sec}$, (c) $20 \text{ cm}^2/\text{sec}$, (d) None of these.

- Q.11. If $y = x^2 + 2x - 1$, then the approximate value of y when x changes from 1 to 0.98 is given by
(a) -0.08, (b) -0.8, (c) 0.8, (d) None of these.
- Q.12. The critical numbers of $f(x) = x + \sin x$ on interval $[0, 2\pi]$ is
(a) 0, (b) π , (c) 2π , (d) None of these.
- Q.13. If $f(x) = 3x^3$ has point of inflection at
(a) 0, (b) 3, (c) 0,3, (d) None of these.
- Q.14. If $f(x) = \tan^{-1}(2x + 3)$, then $f'(0)$
(a) $\frac{1}{5}$, (b) 0, (c) not defined, (d) None of these
- Q.15. The exact value of $\cos\left(\sin^{-1}\left(\frac{3}{5}\right)\right)$ equals
(a) $\frac{3}{4}$, (b) $\frac{4}{5}$, (c) $\frac{5}{4}$, (d) None of these.
- Q.16. The range of the function $f(x) = 2\sin^{-1}(x)$, is
(a) $[-1, 1]$, (b) $[-2, 2]$, (c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $[-\pi, \pi]$
- Q.17. The length of the minor axis of the ellipse $9x^2 + 4y^2 = 36$ is
(a) 9, (b) 4, (c) 6, (d) None of these.
- Q.18. Asymptotes of the hyperbola $4x^2 - 9y^2 = 36$ are
(a) $y = \pm\frac{3}{2}x$, (b) $y = \pm\frac{2}{3}x$, (c) $y = \pm 4x$, (d) None of these.
- Q.19. The parabola $(x-1)^2 = 4(y-2)$ has its focus at the point
(a) (1,2), (b) (2,2), (c) (1,3), (d) None of these.
- Q.20. The conic section $x^2 - 2xy + y^2 - 2x + 3y + 1 = 0$ is a
(a) parabola, (b) ellipse, (c) hyperbola, (d) None of these.

Question.21. If $f(x) = \frac{\sqrt{x-2}+1}{x}$, $g(x) = \frac{1}{x}$, $x \neq 0$, find $(f \circ g) x$ and $(g \circ f) x$.

[6]

Question.22. Find all values of 'a' and 'b' for which the following function is continuous at $x = 1$

$$f(x) = \begin{cases} \frac{(x-1)^2 - a(x-1)}{\sin(x-1)} & \text{if } x < 1 \\ 8 & \text{if } x = 1 \\ \frac{\tan(b(1-x))}{(1-x)} & \text{if } x > 1 \end{cases} \quad [8]$$

Question.23. Find the horizontal and vertical asymptotes for the graph of the function

$$f(x) = \frac{\sqrt{9x^2 - 3x + 5}}{1 - 3x}.$$

[8]

Question.24. Find $\frac{dy}{dx}$, if $y = \tan(2 \cos x + \sin^2 x)^3$.

[6]

Question.25. Find dimensions of a rectangle with area 36ft^2 , such that its perimeter is minimum.

7

[6]

Question.26. If $f(x) = \frac{x+2}{x+1}$, prove that $f(x)$ is one - to one ,
find $f^{-1}(x)$, also find the range of $f(x)$.

[5]

Question.27. Let $f(x) = \sqrt{x}(x-2)$. Find

- (a) Domain of $f(x)$, (b) Critical numbers, (c) Interval of increasing and decreasing, and (d) Local extrema by using First derivative test. **[8]**

Question.28. If $x^2 + y^2 + \sin^{-1}(x + y) = 0$, find $\frac{dy}{dx}$ at point $(0, 0)$. [6]

Question.29.(a) Classify the cone, **(b)** find the rotation equations, **(c)** use rotation equation to eliminate the 'xy' term of the given cone $x^2 - xy + y^2 - 4x + 5 = 0$ [7]