ASSESSMENT OF BRITTLE FAILURE AT THE ULTIMATE FLEXURAL LIMIT STATE OF R.C. BEAMS BY COMPUTER SIMULATION

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Abstract

Reinforced concrete beams at the ultimate flexural limit state may fail by concrete crushing even when they are reinforced below the maximum reinforcement ratio specified by the ACI Code. One of the factors contributing to this uncertainty is the variability of the strength of concrete and reinforcing steel. This paper presents the relationship between probability of brittle failure of reinforced concrete beams at the limit state and the variability in the yield strength of reinforcing steel and compressive strength of concrete. Monte-Carlo simulation is employed to randomly generate strength parameters. Results indicated that probability of brittle failure at the limit state is adversely affected by low quality of concrete and high yield strength of reinforcing steel. A criterion for maximum reinforcement ratio to contain the risk of brittle failure within an acceptable level is proposed.

1 Introduction

The limit state design of reinforced concrete flexural members is based on strain compatibility and force equilibrium. The balanced flexural limit state condition of a beam is the case when the strain of concrete at the extreme compression fiber reaches the ultimate strain at the time when the tension reinforcement reaches yield strain.

According to ACI 318M[1], the balanced reinforcement ratio ρ_b is calculated as,

ρ_b = خطأ! خطأ! (1)

in which f'_c and f_y are in (MPa) and β_1 is a function of f'_c (ACI Code, Section 10.2.7.3).

It is essential to design a reinforced concrete member with sufficient ductility to avoid brittle failure in flexure especially for seismic design. To ensure that the failure of reinforced concrete beams is initiated and proceeded by yielding of tensile steel, the ACI 318M requires that the maximum tensile reinforcement ratio to be $(\rho - \rho') \leq 0.75\ \rho_b$ where ρ and ρ' are the tension and compression reinforcement ratios respectively.

Reinforced concrete sections at the limit state may fail by concrete crushing even when they are reinforced below the maximum reinforcement ratio specified by the ACI Code. One of the factors contributing to this uncertainty is the variability of the strength of concrete and reinforcing steel.

The test results on the flexural behavior of full scale reinforced concrete beams[2] employing reinforcing steel produced in Saudi Arabia indicate that beams have low ductility even with reinforcement ratio lower than the maximum reinforcement ratio as recommended by the ACI 318M[1].

This study investigated the relationship between probability of brittle failure at the flexural limit state of reinforced concrete beams, $P_f(CF)$ and the variability in yield strength of reinforcing steel and compressive strength of concrete. The adverse effect of high tension reinforcement ratios is also investigated. A criterion for maximum reinforcement ratio to contain the risk of brittle failure within an acceptable level is proposed.

2 Previous Studies

The probabilistic behavior of reinforced concrete beams in bending was investigated by Allen[3]. He concluded that a significant proportion of failures are compressive even when the section is under-reinforced according to ACI 318M. The probability of brittle failure at maximum reinforcement ratio reaches 18 percent for low rate construction loading and minimum workmanship[3] .

Ito and Sunikama[4], investigated probabilities of brittle failure of reinforced concrete beams. They concluded that, the higher the mean-to nominal ratio of steel yield stress, λ_s, and its coefficient of variation, V_s, the higher probability of compression failure, Pr(CF). They also reported high values of Pr(CF) at $\rho = 0.75\ \rho_b$. It was found that for site-mixed concrete it is necessary to reduce the maximum reinforcement ratio to $0.35\rho_b$. They also found that if the level of quality control in production of reinforcing bars is upgraded so that V_s is 6 percent, it would be satisfactory to employ $\rho \leq 0.55\rho_b$ for the site-mixed concrete and $\rho \leq 0.6\rho_b$ for the ready-mixed concrete.

3 Simulation of Beam Flexural Behavior

The Monte-Carlo technique is employed for simulation of the flexural behavior of beam sections at the ultimate limit state and for random generation of related parameters such as compressive strength of concrete, yield strength of reinforcing steel and sectional dimensions.

3.1 Input Data for Concrete

3.1.1 Constitutive Model for Concrete

The stress-strain curve for concrete suggested by Hognestad, et al.[5] is employed in the procedure. The curve is presented by a second degree parabola for the ascending part of the relation which can be expressed as:

$$f_{ci} = f_c \left[2\left(\text{خطأ!} \right) - \left(\text{خطأ!} \right)\text{خطأ!} \right] \qquad (2)$$

and a straight line over the descending part which can be expressed as,

$$f_{ci} = f_c \ [1 - z (\varepsilon_{ci} - \varepsilon_{co})] \qquad (3)$$

where f_{ci} is the compressive stress, ε_{ci} is concrete strain, f_c is ultimate concrete compressive strength, ε_{co} is the concrete strain at the ultimate concrete compressive strength which is assumed to be 0.002 and z is the slope of the linear descending part of the relation which reflects the level of concrete confinement. z is usually assumed in the range of 150 for moderate concrete confinement.

3.1.2 Ultimate strain of the concrete

The ultimate strain of the concrete, ε_{cu}, is function of compressive strength and the rate of loading. Under static loads, ε_{cu} was estimated as follows[5],

$$\varepsilon_{cu} = 0.004 - 2.23 \times 10^{-5} \ fc \qquad (4)$$

3.1.3 Statistical Characteristics of Concrete Strength

Two types of concrete were considered in the study: ready-mixed, RM, and site-mixed, SM, concrete. The first type represents the average quality with mean-to-nominal ratio, λ_c, of 1.0, coefficient of variation, V_c, of 20 percent and normal distribution function[6]. The second type represents the poor quality with $\lambda_c = 0.85$, $V_c = 40$ percent and a log-normal distribution function[6].

3.2 Input Data for Reinforcing Steel

3.2.1 Constitutive Model for Steel

The constitutive model that represents the behavior of reinforcing steel over three strain-ranges is expressed as follows,

$$f_s = E_s \varepsilon_s \qquad \text{for} \quad 0 < \varepsilon_s \le \varepsilon_y \qquad (5a)$$

$$f_s = f_y \qquad \text{for} \quad \varepsilon_y < \varepsilon_s \le \varepsilon_{sh} \qquad (5b)$$

$$f_s = f_y + E_{sh} (\varepsilon_s - \varepsilon_{sh}) \qquad \text{for} \quad \varepsilon_s > \varepsilon_{sh} \qquad (5c)$$

in which f_s and ε_s are the steel stress and strain respectively, f_y is the yield strength, ε_y is yield strain, ε_{sh} is the strain at the initiation of strain hardening and E_s and E_{sh} are the steel moduli of elasticity in the elastic and strain hardening ranges, respectively.

3.2.2 Statistical Characteristics of Reinforcing Steel

The study considered high tensile steel (fy = 413 MPa) having a mean-to-nominal ratio of yield strength λ_s = 1.34 , coefficient of variation V_s = 4.3 percent and a normal distribution function[7].

3.3 Input Data for Sectional Dimensions

In general, the variability in sectional dimensions tends to be very small and rather less important than the variability in material parameters. therefore, sectional dimensions b and h are considered to be deterministic whereas the effective depth of the tension and compression reinforcements, d and d' , are considered random variables with λ = 1.0 and coefficient of variation = 2 and 20 percent, respectively[8].

3.4 Simulation Process

A computer program to simulate the beam behavior at limit state and generate the random variables is developed. The random variables are f_c, f_y , ε_{sh}, E_s, E_{sh}, d, and d' whereas the deterministic parameters are ε_{co}, z, ρ, ρ', h and b are assumed to be deterministic. The program includes the following steps:

(1) select b, h, ρ, and type of concrete (RM or SM),

(2) generate the random variables f_c, f_y , e_{sh}, E_s, E_{sh}, d, and d',

(3) calculate ε_{cu} using Eq. 4 and ε_y as f_y/E_s ,

(4) calculate the depth of neutral axis, x, on the basis of strain compatibility and force equilibrium of the beam section,

(5) calculate the strain in steel and check the case of brittle failure, i.e. the case when $\varepsilon_s < \varepsilon_y$ at $\varepsilon_c = \varepsilon_{cu}$,

(6) repeat steps 2 to 5 number of cycles to give reliable result and calculate the probability of brittle failure.

4 Sensitivity Analysis

The variation of probability of compression failure Pr(CF) with λ_s and V_s was investigated. The analysis was performed for RM concrete (f'c = 25 MPa, λ_c = 1.0 and V_c = 20 percent) with reinforcement ratio $(\rho - \rho')/\rho_b = 0.60$. The mean-to nominal ratio λ_s was taken between 1.0 and 1.4 and V_s was taken as 5, 10 and 15 percent.

The variation of Pr(CF) with λ_c and V_c was investigated. The analysis was performed for steel with f_y =413 MPa, λ_s = 1.34 and V_s = 4.3 percent) with reinforcement ratio $(\rho - \rho')/\rho_b = 0.60$. For concrete, λ_c was taken between 0.8 and 1.3 and V_c was taken as 20, 30 and 40 percent .

The variation of Pr(CF) with reinforcement ratio, $(\rho - \rho')/\rho_b$, was investigated. The reinforcement ratio was taken between $0.2\rho_b$ and $0.75\rho_b$. The analysis was conducted for RM and SM concretes.

5 Discussion of Results

Fig. 1 presents the variation of Pr(CF) with λ_s and V_s. Results indicate that Pr(CF) increase with increasing λ_s and V_s . This is mainly attributed to higher yield strain of steel as λ_s increases. The slope of these curves increases with increasing λ_s. These curves allow one to compare Pr(CF) for different sources of steel at reinforcement ratio of $0.6\rho_b$. For example, the Pr(CF) using steel produced in United States is about 2 percent whereas with the Saudi steel, Pr(CF) is about 9 percent.

Fig. 2 presents the variation of Pr(CF) with λ_c and V_c. Results indicate that Pr(CF) increases with decreasing λ_c and increasing V_c. At the limit state with low strength of concrete large depth of neutral axis is needed to maintain the state of the section equilibrium which increases the strain in concrete and reduces the strain in the reinforcement and therefore increases probability of brittle failure of the beam at the limit state.

Fig. 3 presents the variation of Pr(CF) with $(\rho-\rho')/\rho_b$ for RM and SM concretes employing the properties of Saudi steel. Results indicated that Pr(CF) increases with increasing $(\rho-\rho')/\rho_b$. The slope of these curves increases with increasing $(\rho-\rho')/\rho_b$. The Pr(CF) for RM concrete is about zero when $(\rho-\rho')/\rho_b \leq 0.4$. At $(\rho-\rho')/\rho_b = 0.75$, Pr(CF) is about 33% and 55% for RM and SM concretes, respectively. At 10 percent risk of brittle failure, the maximum ratios of $(\rho-\rho')/\rho_b$ are found about 0.6 and 0.4 for RM and SM concretes respectively.

6 Conclusion

Results indicated that probability of brittle failure at the limit state is adversely affected by the low quality of concrete and high yield strength of reinforcing steel. It is also adversely affected with the high reinforcement ratio. A criterion for maximum reinforcement ratio based on specifying the acceptable risk of having brittle failure at the limit state and calculating the ρ_{max} is proposed. Results indicate that at 10 percent risk of brittle failure at the ultimate flexural limit state the maximum reinforcement ratios are found about 0.6 and 0.4 of balanced reinforcement ratio for ready-mixed and site-mixed concretes respectively.

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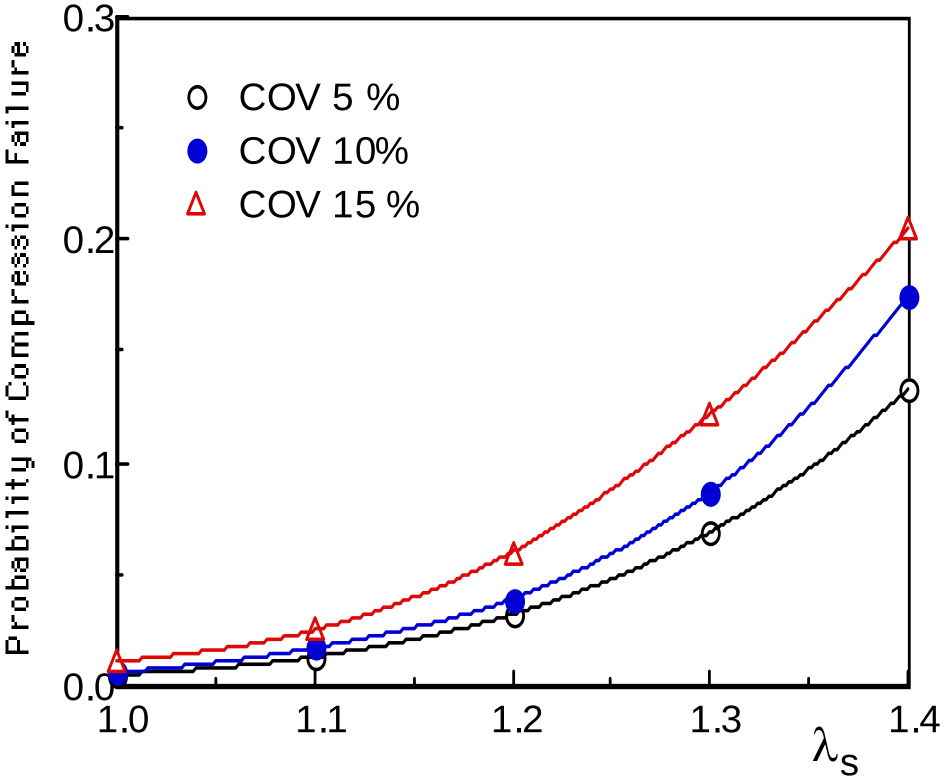

Fig. 1 Variation of the Probability of Compr
Failure with Yield Strength of Steel ρ=0.6ρ_b

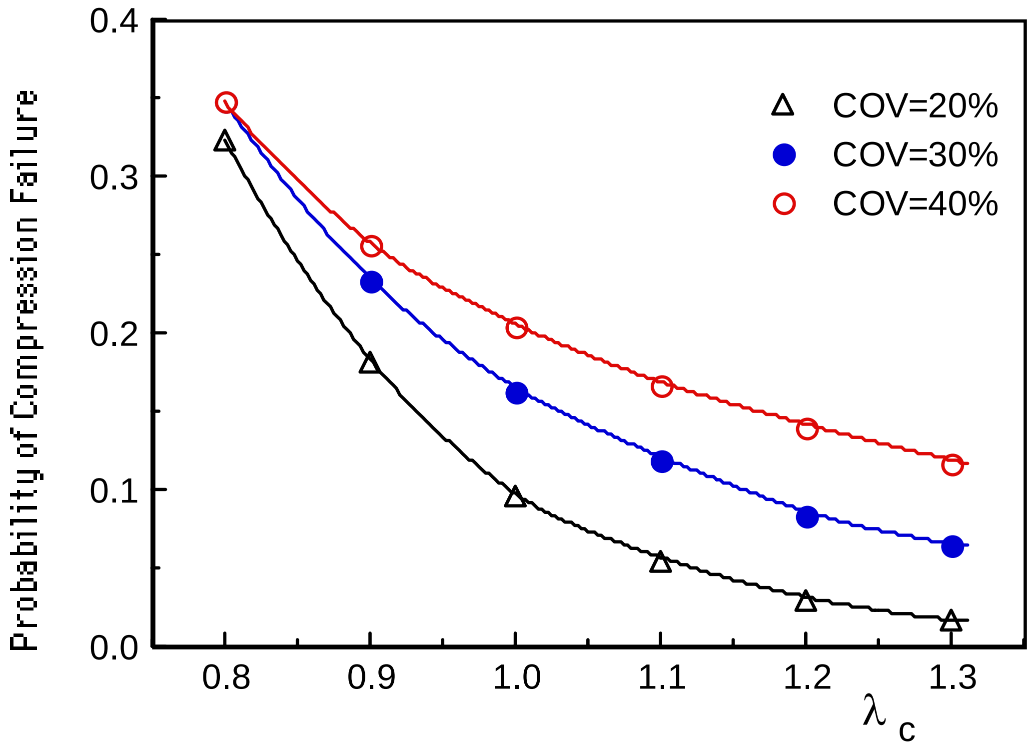

Fig. 2 Variation of the Probability of Brittle Failure with the Mean-to-Nominal Ratio of Compressive Strength of Concrete

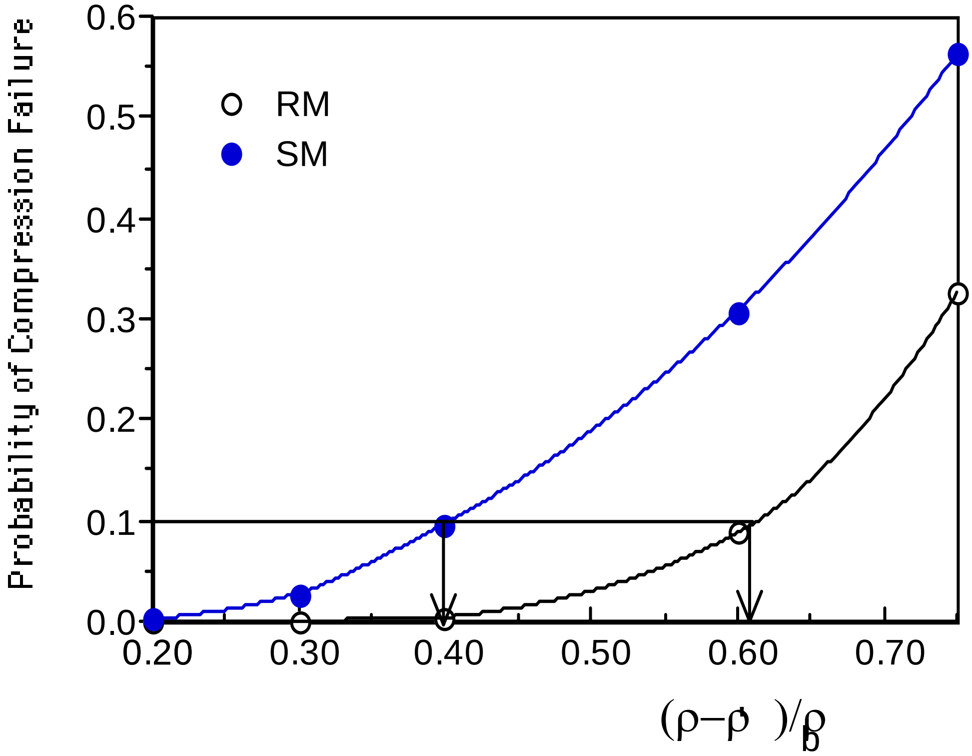

Fig. 3 Variation of the Probability of Compress Failure with Reinforcement Ratio