RELIABILITY-BASED CRITERION FOR MAXIMUM REINFORCEMENT RATIO OF REINFORCED CONCRETE BEAM SECTIONS

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Abstract

This paper presents reliability-based approach for the maximum reinforcement ratio for reinforced concrete flexural members. The study was based on sensitivity analysis of beams at their flexural limit state. The statistical characteristics of strength parameters under the prevailing construction practices in Saudi Arabia are employed. The maximum reinforcement ratio that specified by ACI 318M-95 is critically examined. At the maximum reinforcement and employing local materials (concrete and reinforcement), the probability of brittle flexural failure was found to be higher than that reported in literature. This is mainly attributed to the high yield strength of the reinforcement and low compressive strength of the concrete. Two approaches were proposed to control the probability of brittle failure. The first approach includes replacing the nominal strength values of reinforcement and concrete by the corresponding mean values in the ACI formula for maximum reinforcement ratio. The second approach includes determination of the acceptable probability of brittle failure at the limit state and calculation of the maximum reinforcement ratio from the relationships developed in this study.

Keywords: beams, bending, building code, compressive strength, ductility, failure, probability theory, reinforced concrete, reinforcing steel, and reliability.

RELIABILITY-BASED CRITERION FOR MAXIMUM REINFORCEMENT RATIO OF REINFORCED CONCRETE FLEXURAL MEMEBERS IN SAUDI ARABIA

INTRODUCTION

The limit state design of reinforced concrete flexural members is based on the principles of strain compatibility and force equilibrium. The ultimate balanced flexural strength of a member is reached when the strain in the extreme compression fiber reaches the ultimate strain of concrete at the time the tension reinforcement reaches yield strain. It is essential to design a reinforced concrete member with sufficient ductility to avoid brittle failure in flexure specially for seismic design.

According to ACI 318M-95 [1], Section 10.3.2, the balanced reinforcement ratio ρ_b is calculated as,

$$\rho_b = \frac{0.85\beta_1 f_c'}{f_y}\frac{600}{600+f_y} \qquad (1)$$

in which f'_c is the concrete nominal compressive strength and f_y is the reinforcement nominal yield strength in (MPa) and β_1 is a function of f’c (ACI Code, Section 10.2.7.3).

To ensure that failure of reinforced concrete beams is initiated and proceeded by yielding of tensile steel, the ACI 318M-95, Section 10.3.3 requires that, for non-seismic conditions, the maximum tensile reinforcement ratio to be $(\rho - \rho') \leq 0.75\, \rho_b$ where ρ is the section tension reinforcement ratio, ρ' is the section compression reinforcement ratio and ρ_b is the balanced reinforcement ratio. This criterion ensures that the curvature ductility factor is about 2. For the strain rates of 0.05/s and more, ACI 318M-95, Section 21.3.2.1, limits the tensile reinforcement to $\rho \leq 0.025$ and ACI318M-95, Section 21.3.2.2, requires $\rho' = 0.5\,\rho$.

Reinforced concrete sections at the limit state may fail by concrete crushing even when they are reinforced below the maximum reinforcement ratio specified by the ACI318M-95 Code. One of the factors contributing to this uncertainty is the variability of the strength of concrete and reinforcing steel. The nominal yield strength of steel falls in the lower tail of the probability density function and as such, the actual yield strength, in general, is higher than the specified value. The margin provided by the ρ_{max} does not ensure a ductile failure specially when the mean-to-nominal ratio of yield strength, λ_S, is high.

The minimum reinforcement ratio is essential to prevent early brittle failure of reinforced concrete beams by steel rupture. ACI 318-95M specifies the minimum reinforcement as

$$\rho_{min} = \frac{\sqrt{f_c'}}{4\,f_y} \geq \frac{1.4}{f_y} \qquad (2)$$

This criterion ensures that nominal flexural strength exceeds the cracking moment by a safety factor of 1.5.

The ACI 318M-95 [1] is widely adopted for the design of reinforced concrete structures in Saudi Arabia. Test results on the flexural behavior of full scale reinforced concrete beams reported by Al-Zaid et al. [2] showed that beams with reinforcement even lower than the maximum reinforcement as specified by the ACI 318M-95 [1] have low ductility. The mean-to-nominal ratio of flexural capacity, λ_R, was found to be higher than those suggested by Allen and MacGregor [3, 4].

The prime cause of such behavior was the high mean-to-nominal ratio of the steel yield strength, λ_s .

The Saudi Iron and Steel Company (HADEED) includes a quenching stage in the production process. This process results in bars with a relatively high yield strength. The mean yield strength for Grade 420 bar is 554 MPa. When the maximum reinforcement ratio, ρ_{max}, as defined by ACI 318M-95 is employed with such high values of yield strength the desirable level of beam ductility can not be attained and the probability of brittle failure at the limit state is expected to be very high .

This study presents a sensitivity analysis for the probability of brittle failure, Pr(CF), at the limit state of reinforced concrete beams. Several parameters have been included in the analysis such as: (1) variabilities in the yield strength of reinforcing steel and compressive strength of concrete, (2) tension and compression reinforcement ratios, and (3) strain rate of loading. The results serve as a design guide for selecting the appropriate limits of reinforcement ratio.

PREVIOUS STUDIES

Statistical Characteristics of Concrete

Based on a normal rate of application of the test load (static load), the coefficient of variation ,V_c , of the in-situ compressive strength for concrete grades 35 and 20 MPa are estimated to be 15% and 18%, respectively [4 to 6]. Concrete strength was assumed to follow a normal distribution [4 and 5]. Ellingwood [7] estimated the V_c to be 20.7% under average control. Freudenthal, et al. [8] reported that the distribution of f'_c conformed to a logarithmic normal distribution under poor quality control. Allen [3] reported that values of V_c for concrete compressive strength for minimum and good workmanship is 18 and 15 percent, respectively.

Arafah et al. [9, 10] estimated the statistics of ready-mixed (RM) concrete and at-site mechanically-mixed (SM) concrete under the prevailing concreting practices in Saudi Arabia. The results of 636 strength tests on RM concrete indicated that the mean-to-nominal ratio of concrete compressive strength, λ_c, and the strength coefficient of variation, V_c, are about 1.0 and 20 percent, respectively, and the strength is well represented by the normal distribution. The results of 45 strength tests on SM concrete indicated that λ_c and V_c are about 0.85 and 40 percent, respectively, and concrete strength is well represented by the log-normal distribution. These results were adopted in the this study and listed in Table 1.

The ultimate strain of the concrete, ε_{cu}, is a function of the compressive strength and the rate of loading. Under static loading, ε_{cu} was estimated as follows [11];

$$\varepsilon_{cu} = 0.004 - 2.23 \times 10^{-5} f_c \qquad (3)$$

For test specimens loaded at a rate of loading R (MPa/sec), the concrete compressive strength was represented by a normal distribution [5] with mean value as;

$$f_{cmr} = f_{cm} [0.89 (1.173 + 0.08 \log_{10} R)] \qquad (4)$$

where f_{cmr} is the mean concrete strength at R rate of loading (MPa/sec), and f_{cm} is the mean concrete strength under static loading. The mean ultimate strain of the concrete under earthquake loading, ε_{cu}, was assumed as [3];

$$\varepsilon_{cum}= 0.0034 -1.88 \times 10^{-5} f_c \qquad (5)$$

with coefficient of variation of 15 percent. Equations 3 to 5 were adopted in this study.

Statistical Characteristics of Reinforcing Steel

Mirza and MacGregor [12] indicated that the mean-to-nominal ratio, λs, and the coefficient of variation , V_s, of yield strength for Grade 420 steel were 1.11 and 9.8%, respectively. Ito and Sumikama [13] studied typical statistics of the reinforcement yield strength, f_y (Grade 420). The maximum mean value was 486 MPa with corresponding V_S of 10.5 percent. Allen [3] employed λ_s as 1.1 and 1.18 for low rate and earthquake rate of loading respectively.

Arafah et al. [9] studied 625 test results of Grade 420 bars. Results indicated that λ_S was 1.22 and V_S was about 10 percent. Al-Behairi [14] investigated the probabilistic characteristics of steel bars produced through the bar quenching process, and found that λ_S and V_S are 1.34 and 4.3 percent, respectively, and the yield strength is well represented by the normal distribution function. These statistics were adopted in this study and are listed in Table 1.

The reinforcement yield strength significantly increases at higher rates of loading. In this study, the effect of strain rate, r, on the yield strength is introduced by the following formulas [15];

$$f_{yr} = 21.35 + 1.2\ f_y + (\ 4.48 + 0.05\ f_y\ )\ \log_{10} r\ \geq\ f_y \qquad (6)$$

where f_{yr} is yield strength of steel (MPa) under a specified strain rate .

Statistical Characteristics of Sectional Dimensions

The deviation of sectional dimension parameters from their nominal values affects the behavior of beam sections. In general, the variability in sectional dimensions tends to be very small and rather less important than the variability in material parameters. Based on the test results in [9, 16] the coefficient of variation of the depth of reinforcement in the tension and compression regions are taken as 2 and 20 percent, respectively.

Behavior of Reinforced Concrete Beams

The probabilistic behavior of reinforced concrete beams in bending was investigated by Allen [3]. It was concluded that the probability of brittle failure at maximum reinforcement ratio reaches 18 percent for low rate construction loading and minimum workmanship. Higher probabilities were obtained for 1-sec earthquake rate of loading [3].

The effect of steel reinforcement ratio on resistance factor, ϕ, for reinforced concrete beams in flexure was investigated by MacGregor [17] and found that, in order to maintain constant reliability of a beam in flexure, the level of ϕ drops significantly when ρ/ρ_b exceeds 0.5. This is because as ρ approaches ρ_b the probability of compression failure in beams increases. It was proposed that the value of ρ/ρ_b be limited to 0.6.

Park and Dai [18] investigated the curvature ductility factor, the ratio of the curvature at first yield of reinforcing steel to that at ultimate state, $\mu_\phi = \phi_u/\phi_y$ and concluded that the general requirement $(\rho - \rho') \leq 0.75\ \rho_b$ ensures a curvature ductility factor of more than 2 and the requirement $(\rho - \rho') \leq 0.50\ \rho_b$ of more than 4.

Al-Haddad, M. [19] studied the effect of reinforcement ratio on the curvature ductility factor and concluded that the ACI 318M-95 provisions of limiting maximum longitudinal steel ratio do not ensure sufficient ductility for conventional and seismic designs when used with Saudi steel and concrete.

Ito and Sunikama [13], investigated the effect of the reduction coefficient of the balanced steel ratio on the probabilities of compression failure of reinforced concrete beams and found that for site-mixed concrete having V_c of 20 percent and ready-mixed concrete having V_c of 15 percent, it is necessary to limit ρ to $0.35\rho_b$. It was also found that if the level of quality control in production of reinforcing bars is upgraded so that V_s is 6 percent, it would be satisfactory to limit ρ to $0.55\rho_b$ for the site-mixed concrete and $0.6\rho_b$ for the ready-mixed concrete.

ANALYTICAL MODEL AND ASSUMPTIONS

Constitutive Model for Concrete

The stress-strain curve for concrete suggested by Hognestad et al. [11] is employed in the procedure. As shown in Fig. 1, the curve is presented by a second degree parabola for the ascending part of the relation which can be expressed by:

$$f_{ci} = f_c \left[2\left(\frac{\varepsilon_{ci}}{\varepsilon_{co}}\right) - \left(\frac{\varepsilon_{ci}}{\varepsilon_{co}}\right)^2 \right] \qquad (7)$$

and a straight line over the descending part which can be expressed by;

$$f_{ci} = f_c \left[1 - z\left(\varepsilon_{ci} - \varepsilon_{co}\right) \right] \qquad (8)$$

where f_{ci} is the compressive stress, ε_{ci} is concrete strain, f_c is the ultimate concrete compressive strength, ε_{co} is the concrete strain at the ultimate concrete compressive strength which is assumed to be 0.002 and z is the slope of the linear descending part of the relation which reflects the level of concrete confinement. z is usually assumed between 100 and 150 for moderate concrete confinement. Linear brittle stress-strain relation for concrete in tension with a rupture strain equal to f_r/E_c is employed.

Constitutive Model for Steel

The model expresses the constitutive behavior over the three strain-ranges as,

$$f_s = E_s\,\varepsilon_s \quad \text{for} \quad 0 < \varepsilon_s \le \varepsilon_y \qquad (9a)$$

$$f_s = f_y = E_s\,\varepsilon_y \quad \text{for} \quad \varepsilon_y \le \varepsilon_s \le \varepsilon_{sh} \qquad (9b)$$

$$f_s = f_y + E_{sh}\,(\varepsilon_s - \varepsilon_{sh}) \quad \text{for} \quad \varepsilon_s \ge \varepsilon_{sh} \qquad (9c)$$

in which f_s and ε_s are the reinforcement stress and strain respectively, f_y and ε_y are the reinforcement yield strength and strain respectively, ε_{sh} is the strain at the initiation of strain hardening and E_s and E_{sh} are the steel modules of elasticity and modules of strain hardening, respectively.

Monte- Carlo Technique for Simulation of Section Behavior

The Monte-Carlo technique is employed for simulation of the random variables and the behavior of the beam sections. Based on the statistics given in Table 1, the computer program simulates the random variables f_c, f_y, ε_{sh}, E_s, E_{sh}, d, and d' whereas the parameters ε_{co}, z, A_s, A_s', h and b are assumed to be deterministic parameters. Strength parameters are shown in Fig. 3. The program includes the following steps:

(1) select b, h, ρ, and type of concrete (RM or SM),

(2) generate the random variables f_c, f_y , ε_{sh}, E_s, E_{sh}, d, and d',

(3) calculate ε_{cu} using Eq. 3 and ε_y as f_y/E_s ,

(4) calculate the depth of neutral axis, x, on the basis of strain compatibility and force equilibrium of the beam section,

(5) calculate the strain in steel,

(6) check the case of compression failure (the case when $\varepsilon_s < \varepsilon_y$ at $\varepsilon_c = \varepsilon_{cu}$),

(7) check the case of steel rupture failure (the case when $\varepsilon_s > \varepsilon_{su}$ at $\varepsilon_c = \varepsilon_{cu}$),

(8) repeat steps 2 to 7 thousand cycles and calculate the probability of compression failure or the probability of steel rupture depending on the reinforcement ratio.

SENSITIVITY ANALYSIS

The variation of probability of compression failure Pr(CF) with λ_s and V_s was investigated. The analysis was performed for RM concrete (f'_c = 25 MPa, λ_c = 1.0 and V_c = 20 percent) with reinforcement ratio $(\rho-\rho')/\rho_b = 0.60$. λ_s was taken between 1.0 and 1.4 and V_s was taken as 5, 10 and 15 percent as shown in Fig. 4.

The variation of Pr(CF) with λ_c ,and V_c was investigated. The analysis was performed for Saudi steel (f_y =413 MPa, λ_s = 1.34 and V_s = 4.3 percent) with reinforcement ratio $(\rho-\rho')/\rho_b = 0.60$. For concrete, λ_c was taken between 0.8 and 1.3 and V_c was taken as 20, 30 and 40 percent as shown in Fig. 5.

The variation of Pr(CF) with reinforcement ratio, $(\rho-\rho')/\rho_b$, was investigated. The reinforcement ratio was taken between $0.2\rho_b$ and $0.75\rho_b$. The analysis was conducted for RM and SM concretes with Saudi steel as shown in Fig. 6.

The variation of probability of steel rupture Pr(SF) at the limit state with reinforcement ratios ρ lower than ρ_{min} as specified by ACI 318M was investigated. Reinforcement ratio $(\rho-\rho')/\rho_b$ was taken between 0.02 and 0.12 as shown in Fig. 7

The variation of Pr(CF) with reinforcement ratio was performed at earthquake rate of loading assumed to correspond to a strain rate of 0.05/s. The mean compressive strength of concrete and yield strength of steel were calculated using Eqs. 2 and 5 respectively. The mean value of ε_{cu} was calculated using Eq. 4. The analysis was conducted for both RM and SM concretes as shown in Fig. 8.

RESULTS, ANALYSIS AND DISCUSSION

Fig. 4 presents the variation of Pr(CF) with λ_s and V_s. Results indicate that Pr(CF) increase with increasing λ_s and V_s . This is mainly attributed to higher yield strain of steel as λ_s increases. The slope of these curves increases with increasing λ_s. These curves allow one to compare Pr(CF) for different sources of steel at reinforcement ratio of $0.6\rho_b$. For example, the Pr(CF) using steel produced in United States is about 2 percent whereas with the Saudi steel, Pr(CF) is about 9 percent.

Fig. 5 presents the variation of Pr(CF) with λ_c and V_c. Results indicate that Pr(CF) increases with decreasing λ_c and increases with increasing V_c. At low strength of concrete, either due to low λ_c or high V_c , and to maintain the equilibrium of the section at the limit state the depth of neutral axis is large which causes a considerable reduction in the strain of the tension steel and increases probability of brittle failure.

Fig. 6 presents the variation of Pr(CF) with $(\rho-\rho')/\rho_b$ for RM and SM concretes employing the properties of Saudi steel. Results indicated that Pr(CF) increases with increasing $(\rho-\rho')/\rho_b$. The slope of these curves increases with increasing $(\rho-\rho')/\rho_b$. The Pr(CF) for RM concrete is about zero when $(\rho-\rho')/\rho_b \leq 0.4$. At $(\rho-\rho')/\rho_b = 0.75$, Pr(CF) is about 33% and 55% for RM and SM concretes, respectively, .

Fig. 7 presents the variation of Pr(SF) with $(\rho-\rho')/\rho_b$ for RM and SM concretes. Results indicate that Pr(SF) increases with decreasing $(\rho-\rho')/\rho_b$. The values of Pr(SF) are close to zero for values of $(\rho-\rho')/\rho_b$ higher than 0.08. The ACI specified values of ρ_{min} for RM and SM concretes are $0.14\rho_b$ and $0.16\rho_b$, respectively, which are conservative for the Saudi steel.

Fig. 8 presents the variation of Pr(CF) with $(\rho-\rho')/\rho_b$ at earthquake rate of loading for RM and SM concretes employing the properties of Saudi steel. Results indicated that values of Pr(CF) are higher than those obtained in case of low rate of loading. This is mainly attributed to the high yield stress and strain of steel at that high rate of loading and due to the low ultimate strain of concrete at high rate of loading.

CRITERIA FOR MAXIMUM REINFORCEMENT RATIO

Based on the analysis performed, two approaches were proposed to contain the Pr(CF) and ensure the ductility of R.C. beams at the limit state. The first approach is to account for the variations in the yield strength of steel and compressive strength of concrete in ACI formula. The second approach is to specify an acceptable probability of brittle failure (say 10 percent) and determine the maximum reinforcement ratio accordingly.

Regarding the first proposed approach, the ACI 318M-95 definition of balanced reinforcement ratio is based on the nominal values of the yield strength of steel and the compressive strength of concrete which are in the lower tail of the corresponding strength distribution function. The actual values of the steel yield strength are much higher than the nominal value. The approach is based on replacing nominal strengths of concrete and reinforcement by their respective mean values. These values account for the actual variations of the steel and concrete strengths. The modified balanced reinforcement ratio, ρ_{bm}, becomes;

$$\rho_{bm} = \frac{0.85 \beta_1 \lambda_c f_c'}{\lambda_s f_y} \frac{600}{600 + \lambda_s f_y} \qquad (10)$$

where λ_c is the mean-to-nominal ratio for concrete which is about 1.0 in the United States whereas it is about 1.0 and 0.85 for RM and SM concretes, respectively, in Saudi Arabia. The mean-to-nominal ratio for steel, λ_s, is about 1.12 in the United States and about 1.34 in Saudi Arabia. This approach reduces the value of balanced reinforcement ratio and increases the ductility of reinforced concrete beams.

This approach has been employed by the ACI 318M-95 in several sections of the seismic design provision. For example, Sections 21.3.4 and 21.4.5 requires to use a factor of 1.25 for the reinforcement yield strength, f_y, in calculating the design forces for shear strength of beams and columns. Section 21.5.1 specifies the same factor for the joint design. The same factor is included the Eq. 21-5 of the ACI 318M-95 for calculating the development length of bars in tension.

The effect of employing ρ_{bm} instead of ρ_b was investigated. Figs. 9 and 10 present the variation of the Pr(CF) with $(\rho-\rho') /\ _{bm}$ for static and earthquake rates of loading considering the RM and SM concretes. The maximum reinforcement ratio ($0.75\ \rho_{bm}$) provides reasonable values of Pr(CF) which are equal to 2 and 12 percent for RM and SM concrete, respectively, at static loads. The corresponding values for dynamic loads are 10 and 26 percent, respectively.

The advantage of this approach is that it accounts for the variation of the reinforcement yield strength and the level of quality control of concrete production. Substituting λ_c =1.0 and λ_s =1.0 the equation returns to the original ACI formula. However, at $\rho = \rho_{max}$, this approach does not provide the same level of risk of brittle failure for different cases of design.

The second proposed approach is based on specifying the acceptable risk of having brittle failure at the limit state (say 10 percent) and calculating the maximum ρ/ρ_b from the relationships that developed in this study. For design purposes in Saudi Arabia, the maximum reinforcement ratios , are found about 0.6 and 0.4 of ρ_b for RM and SM concretes, respectively, as shown in Fig. 6. The corresponding values for dynamic loads are about 0.5 and 0.3, respectively, as shown in Fig. 8 . The advantage of this approach is that the Pr(CF) is constant for all cases of design. However, four factors for maximum reinforcement ratio should be included in the design code which might complicates the design process.

CONCLUSIONS

In this paper, the maximum steel ratio specified by ACI 318M is critically examined employing the statistics of Saudi steel and concrete. It is shown that the probability of brittle failure of beam sections at the limit state employing the local materials is higher than that reported in the literature. Two solutions to contain the probability of brittle failure at the limit state were proposed.

The first approach to contain the probability of compression failure at the section limit state is to replace nominal strengths of concrete and reinforcement in the

ACI 318M-95 criterion for the balanced reinforcement ratio by their respective mean values. This is accomplished by multiplying the nominal concrete and reinforcement strengths by their corresponding mean-to-nominal ratios. This approach reduces the probability of compression failure at the limit state from 33% to about 2 %.

The second approach is to reduce the maximum reinforcement ratio as defined by the ACI 318M-95 such that the probability of brittle failure at the limit state is limited to a specified acceptable risk of 10%. From the relationship that developed in this study, the maximum reinforcement is limited to 40% and 60% of the balanced reinforcement for the ready-mixed and site-mixed concretes. The ACI criterion for minimum reinforcement ratio is recommended to be adopted in the Saudi design code even though it is found to be highly conservative.

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List of Notations

A_s: the area of tension reinforcement
A_s': the area of compression reinforcement
d: the effective depth of the beam section
d' : the depth of the compression reinforcement
b: the width of the beam section
h: the depth of the beam section
f_{cm}: the mean concrete strength under static loading.
f_{cmr}: the mean concrete strength at R stress rate of loading
Pr(CF): Probability of compression failure at the limit state.
V_c : the coefficient of variation of concrete compressive strength
V_s : the coefficient of variation of reinforcement yield strength
f'_c : the nominal compressive strength of concrete
f_y : the nominal yield strength of reinforcement.
f_{yr}: theyield strength of steel under a specified strain rate (r).
f_r: concrete modulus of rupture
E_c : concrete modulus of elasticity
E_{sh} : concrete modulus of strain hardening
x: the depth of the neutral axis.
z: the slope of the linear descending part of the concrete stress-strain relationship
λ_s: the mean-to-nominal ratio of the reinforcement yield strength
λ_c: the mean-to-nominal ratio of the concrete compressive strength
λ_R: the mean-to-nominal ratio of flexural capacity
ε_{cu}: the ultimate strain of the concrete
ε_{cum}: the mean ultimate strain of the concrete
ε_{co}: the strain of the concreteat its ultimate strength
ε_y: the yield strain of reinforcement
ε_{sh}: the strain at the initiation of strain hardening of reinforcement
ρ: the section tension reinforcement ratio.
ρ' : the section compression reinforcement ratio
ρ_{min} : the minimum reinforcement ratio
ρ_{max} : the minimum reinforcement ratio
ρ_b : the balanced reinforcement ratio
ρ_{bm}: the modified balanced reinforcement ratio,
ϕ: resistance factor

Table 1. Statistical characteristics of Strength Parameters

Variable	Nominal Value	Mean Value	V %	CDF
Concrete f_c (RM) (MPa) fc (SM) (MPa)	24 20	24 17	20 40	Normal L og-Normal
Steel f_y (MPa) E_s (MPa) E_{sh} (MPa) ε_{sh}	 413 200000 -- --	 554 214505 2920 0.02	 4.3 2.1 16.6 20	Normal
Depth to Steel d (mm) d' (mm)	570 5 0	570 5 0	2 20	Normal

تطوير صيغة تعتمد على أسس ا وثوقية للنسبة القصوى ديد التسليح

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جامعة ا لك سعود - الرياض

ا لخص

يقدم هذا البحث صيغة تعتمد على أسس نظرية ا وثوقية للنسبة القصوى ديد التسليح الكمرات ا رسانية ا سلحة وذلك عند استخدام حديد التسليح و ا رسانة ا صنعة ا ملكة. ويتضمن البحث دراسة احتمالية الا يار غ ا رن الكمرات عند ا الة ا دية لعزم الث للتباين قيم عناصر ا قاومة ونسبة حديد التسليح. كما تشمل الدراسة اختبار القيمة القصوى لنسبة حديد التسليح ا عتمدة مواصفات ا عهد الأمريكي للخرسانة. حيث وجد أنه عند استخدام تلك الصيغة مع خصائص ا واد ا صنعة لياً فإن احتمالية الا يار غ ا رن أعلى من تلك القيم ا عروفة عا ياً وذلك لارتفاع مقاومة ا ضوع ديد التسليح و ا فاض مقاومة الضغط ا رسانة . وقد اقتراح أسلوب للحد من حدوث الا يار غ ا رن الكمرات ا رسانية ا سلحة. الأسلوب الأول يتضمن استبدال قيم ا قاومة الا ية للخرسانة وحديد التسليح با توسط الفعلي لتلك القيم وذلك الصيغة ا عتمدة مواصفات ا عهد الأمريكي. والأسلوب الثا يتضمن ديد احتمالية مقبولة للا يار غ ا رن وحساب القيم القصوى لنسبة حديد التسليح على هذا الأساس من العلاقات ال تطويرها هذه الدراسة.