Problem sheet 2

Section3.1:

Q1: Let $X = (0,\infty)$ and $\tau = \{U \subseteq X; U = \phi, or \{7,10\} \in U\}$

a) Show that τ is a topology on X.
b) Find a bace for this topology.
c) Describe closed sets in this topology.
d) Find, $Int(\aleph), cl(\aleph), Bd(\aleph)$. Where $\aleph$ is the set of natural numbers.
e) Let $A = (1,5) \subset X$, show that the subspace topology (A, τ_A) is the discrete topology. Is $\tau_A \subset \tau$? Verify.
f) Let $B = (2,15)$, describe the subspace topology τ_B, then show that $\tau_B \subset \tau$.

Q2: Let (X,τ) be a topological space, $A \subset X$, show that:

a) If $F \subseteq A$, and F is τ closed, then F is τ_A closed.
b) If $B \subseteq A$, then y is a limit point of B in (X,t) iff y is a limit point of B in (A,τ_A).

Q3: Let $X \neq \phi$, and $A \subseteq X$. Define a topology on X by $\tau = \{X \; or \; B \subseteq X : B \cap A = \phi\}$. Describe τ_A.

Q4: Let $(X.\tau)$ be a topological space, and let $B \subseteq A \subseteq X$. Show that x_0 is a limit point of B in $(X.\tau)$ iff x_0 is a limit point of B in $(A.\tau_A)$.

Section3.2:

Q1: Let $\tau = \{\Re, \phi, (-\infty,7) \cup (7,\infty), \{7\}\}$ be a topology on $\Re$. Let $f : (\Re,\tau) \to (\Re, \mathcal{U})$ be the identity map. Is f a continuous map?

Q2: Let $(X,\tau),(Y,\eta)$ be ropological spaces. Let $f : (X,\tau) \to (Y,\eta)$. Show that:

a) f is closed iff $cl(f(A)) \subseteq f(cl(A)), \forall A \subseteq X$.
b) f is continuaus iff $(f^{-1}(B)) \subseteq f^{-1}(cl(B)), \forall B \subseteq Y$.

Q3: Let $f : (X,\tau) \to (Y,\eta)$. Prove that f is continuous $\Leftrightarrow$ (If N is a nbh of $f(x)$ where $x \in X$, then $f^{-1}(N)$ is a nbh of x in X) $\Leftrightarrow$ for $B \subseteq Y, f^{-1}(IntB) \subseteq Int(f^{-1}(B))$.

Q3: Let $\tau = \{A \subseteq \Re; A = \Re \; or \; 2 \notin A\}$. Let $B \subset \Re$ such that $2 \notin B$. Show that $i : (B,\tau_B) \to (\Re,\tau)$ is open.

خطأ!

$\tau = \{A \subseteq \mathbb{R} : A = \mathbb{R}$ or $\{1,2,3\} \cap A = \phi\}$

Section3.3:

Q1: Let $(\Re,\tau)$ be a topological space, where $\tau = \{A \subseteq \Re; A = \Re \; or \; \{1,2,3,\} \not\subset A\}$. is not topology, why?

~~Prove that $(\Re,\tau)$ is homeamorfhic to $(\Re,\mathcal{U})$~~, where $\mathcal{U}$ is the usual topology on $\Re$.

is $(\mathbb{R}, \tau)$ homeamorphic to $(\mathbb{R}, \mathcal{U})$