Problem sheet 3

Section4.1:

Q1: Let $(X,\tau),(Y,\xi)$ be any two topological spaces, $A \subseteq X, B \subseteq Y$. Show that :

a) $\operatorname{int}(A \times B) = \operatorname{int}(A) \times \operatorname{int}(B)$.

b) If $A \in \tau,\ B \in \xi$, show that $A \times B$ is open in the product topology.

Section6.1:

Q1: Let X be an infinite set, and $x_0 \in X$. Let $\tau = \{A \subset X : A = \phi \, or \, x_0 \in A\}$ be a topology on X .

a) Is (X,τ) a Hausdorff space?

b) Is (X,τ) a compact space?

Q2: Let X be any infinite set with two topologies τ_1 and τ_2 such that (X,τ_2) is a compact space, and $\tau_1 \subset \tau_2$.

a) Show that (X,τ_1) is compact.

b) If $(X,\tau_1)$is Hausdorff, then show that $\tau_1 = \tau_2$.

Q3: Let $X = \Re,\ \tau = \{U \subseteq \Re; U = \Re \ or \ U = \phi, \ or \ U = (a,\infty),\ a \geq 0\}$

a) Is $(\Re,\tau)$ a Hausdorff space.

b) Is $(\Re,\tau)$ a compact space.

Q4: A topological space (X,τ) is called completely Hausdorff space if for ant $x, y \in X,\ x \neq y$ there exist a continues function $f : (X,\tau) \to ([0,1], \mathcal{U}_{[0,1]})$, such that $f(x) = 0,\ f(y) = 1.$ Show that every completely Hausdorff space is a Hausdorff space.

Q5: Prove or disprove:

- Every subspace of a compact topological space is compact.
- The infinite intersection of compact closed sets is closed.
- Every closed subset of a Hausdorff space is compact.

Section6.2:

Q1: Let $(X,\tau),(Y,\xi)$ be cofinite topological spaces, where X,Y are infinite, and let $f:(X,\tau)\to(Y,\xi)$ be any map.

a) Show that if f is onto then f is an open function.

b) Show that if f is onto , one to one then X and $f(X)$ are homeomorphic spaces.

Q2: Let $f:([a,b],\mathcal{U}_{[a,b]})\to(\Re,\mathcal{U})$ be any continuous function. Show that f is uniformly continuous.

Q3: Let (X,τ) be a compact topological space, (Y,ξ) ia any topological space, and if $g:(Y,\xi)\to(X,\tau)$ is open and byjective. Show that (Y,ξ) is compact.

Q4: Let (X,τ) be a compact topological space, (Y,ξ) be Hausdorff and $f:(X,\tau)\to(Y,\xi)$ be a continues injective function. Show that X, $f(X)$ are homeomorphic.