Problem sheet 1

Section2:

Q1: Show that $\{B : A \subseteq B \, or \, B = \phi\}$is a topology on X . If $A \subseteq X$, What topology results when $A = \phi$? when $A = X$.

Q2:Show that the intersection of a family of topologies on X is a topology on X.

Q3: Let Z be the set of positive integers. For each positive integer n let $U_n = \{n, n+1, n+2, ...\}$. Let $\tau = \{\phi, U_1, U_2, U_3, ...\}$. Show that τ is a topology on Z^+. Thin list open sets containing the positive integer 7.

Q4: Prove that τ is the discrete topology on X if and only if every singleton in X is an open set.

Q5: Show that if X is a finite set, then the discrete topology on X coincide with the cofinite topology on X .

Q6: (a) Let X be a nonempty set. Define Define a subset U of X such that $X - U$ is countable, let $\tau = \{X, \phi, \, all \, such U\}$. Show that τ is topology on X . This topology is called the cocountable topology.

(b) If X is countable, describe the topology.

Q7:Give an example of tow topologies on a set X such that their union is not a topology on X .

Q8: Give an example of a collection of open sets in a space X whose intersection is not open.

Q9: Let τ be a topology on X consisting of fore elements, i.e. $\tau = \{X, \phi, A, B\}$ where A and B are non emptyproper subset of X . What conditions must A and B satisfy?

Section 3:

Q1: Show that if U is open in X and A is closed, then $U - A$ is open , and $A - U$ is closed.

Q2: Give an example of a collection of closed sets whose unuion is not closed.

Q3: Let $\mathbb{R}$ have the cofinite topoleg. Describe closed sets in this topology.

Q4: In $\mathbb{R}$, do the rationals form an open set? Closed set? Nither? Both? Prove your assertion

Section 4:

Q1: Consider the set of real numbers, $A=\{x:0<x<1\}\cup\{2,3\}$. Describe $cl(A)$ for the following topologies on $\mathbb{R}$

(a) Usual.

(b) Cofinite.

(c) L.R..

(d) Discrete.

Q2: Let $A\subseteq X$ be a subset of a topological space (X,τ). Show that $Ext(A)=Int(X-A)$.

Q3: Prove or disprove: For any topology on $\mathbb{R}$ 1 is a limit point of $[0,1)$.

Q4: Let $\Im=\{(-\infty,a):a\in\Re\}$. Show that $\Im$ is a topology on $\Re$. For this topology :

(a) Describe closed sets.

(b) If $A\subseteq\Re$, find $Cl(A)$.

(c) If $A\subseteq\Re$, find $Int(A)$.

Section 5:

Q1: Let $\Im=\{A\subseteq\Re, A=\phi\, or\, 5\in A\}$

(a) Show that $\Im$ is a topology.

(b) Give a base of $\Im$ different than $\Im$ itself.

(c) If $B=\left\{1,2,\frac{1}{2},5,7,0\right\}$, find $cl(B)$. $In(B)$.

(d) If A is any subset of $\Re$, show that :

- $5\in Int(A)\Leftrightarrow A\, is\, open$.
- $Int(A)=\phi\Leftrightarrow 5\notin A$.

Q2: Let $\mathrm{B}=\{A\subseteq\Re : A\, is\, finite\}$. Show that B is a base for a topology on $\Re$. Discribe this topology.