201 math

① Find the limite

$$\lim_{(x,y)\to(0,0)}\left(\frac{xy}{\sqrt{x^2+y^2}}+3e^{xy}\right) = 0+3e^{0} = 3, \text{ since:}$$

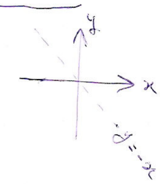
note that $x^2+y^2 \geq x^2 \Rightarrow \sqrt{x^2+y^2} \geq |x|$

$$\Rightarrow \left|\frac{x}{\sqrt{x^2+y^2}}\right| \leq 1$$

$$\Rightarrow 0 \leq \left|\frac{xy}{\sqrt{x^2+y^2}}\right| \leq |y|$$

So, by the sandwich theorem, $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0$.

② Let $f(x,y) = \begin{cases} \dfrac{xy^2}{x^3+y^3} & y \neq -x \\ 0 & y = -x \end{cases}$



a- show that $f_x(0,0)$, $f_y(0,0)$ are exist.

$$f_x(0,0) = \lim_{h\to0} \frac{f(0+h,0)-f(0,0)}{h} = \lim_{h\to0} 0 = 0 \quad \text{also, } f_y(0,0)=0$$

$$f_x(1,1) = \lim_{h\to0} \frac{f(1+h,1)-f(1,1)}{h} = \lim_{h\to0} \frac{\frac{(1+h)}{(1+h)^3+1} - \frac{1}{2}}{h} \cancel{= \lim_{h\to0} \frac{2(1+h)}{h}}$$

$$= \lim_{h\to0} \frac{2(1+h)-(1+h)^3-1}{2h[1+(1+h)^3]} = \lim_{h\to0} \frac{2+2h-1-3h-3h^2+h^3-1}{2h[1+(1+h)^3]}$$

$$= \lim_{h\to0} \frac{\overset{h(-1-3h+h^2)}{-h-3h^2+h^3}}{2h[1+(1+h)^3]} = \lim_{h\to0} \frac{-1-3h+h^2}{2[1+(1+h)^3]} = \frac{-1}{2(2)} = \frac{-1}{4}.$$

- f is not differentiable at $(0,0)$

  يمكن اثباتها كما في التمارين باثبات أن $\lim \varepsilon \neq 0$ مثلا

- f is differentiable at $(1,1)$, since:

  $$f_x(x,y) = \frac{y^5 - 2x^3y^2}{(x^3+y^3)^2} \quad \text{also: } f_y = - - -$$

  and they are continuous on the rectangular region R that contains $(1,1)$, and so, differentiable on R, In particular at $(1,1)$

  (see theorem 12.17 page 1014)