11. Let $A(x)$ be an open sentence with variable x.

a. Prove theorem 1.3.2 (a).

to Prove that $[(\exists! x)A(x) \Rightarrow (\exists x)A(x)]$

let $(\exists! x)(A(x)) \Rightarrow$ the truth set of $A(x)$ is exactly one element
$\Rightarrow$ the truth set of $A(x)$ is non empty
$\Rightarrow (\exists x)A(x)$

b. show that the converse of theorem 1.3.2 (a) is false.

the converse of it, is: $(\exists x)(A(x) \Rightarrow (\exists! x)A(x)$

it is false if the truth set of $A(x)$ is more then one element

for example: $(\exists x)(x^2 \geq 0) \Rightarrow (\exists! x)(x^2 \geq 0)$ it is false

7. Suppose a, b, c and d are positive integers. Write a proof of each biconditional statement.

(a) ac divides bc if and only if a divides b.

let ac divides $bc \Leftrightarrow bc = kac$, $k \in \mathbb{Z}$, $a \neq 0$, $b \neq 0$, $c \neq 0$.
$\Leftrightarrow b = ka$
$\Leftrightarrow a$ divides b.

(c). a is odd if and only if $a+1$ is even

a is odd $\Leftrightarrow a = 2j+1$, $j \in \mathbb{Z}$
$\Leftrightarrow a+1 = 2j+2$, $j \in \mathbb{Z}$
$\Leftrightarrow a+1 = 2(j+1)$, $j \in \mathbb{Z}$
$\Leftrightarrow a+1$ even.

(d). $a+c=b$ and $2b-a=d$ if and only if $a=b-c$ and $b+c=d$.

$a+c=b \wedge 2b-a=d$
$\Leftrightarrow c=b-a \wedge b+\underline{b-a}=d$
$\Leftrightarrow c=b-a \wedge b+c=d$.
$\Leftrightarrow a=b-c \wedge b+c=d$.

9. Prove by contradiction that if n is a natural number, then $\frac{n}{n+1} > \frac{n}{n+2}$.

suppose $\frac{n}{n+1} \leq \frac{n}{n+2} \Rightarrow n(n+2) \leq n(n+1)$
$\Rightarrow n^2+2n \leq n^2+n$
$\Rightarrow 2n \leq n$
$\Rightarrow n \leq 0$, $n \in \mathbb{N}$
a contradiction.