PHYS 580
Nuclear Structure

Chapter5-Lecture1

γ GAMMA DECAY

May 23rd 2007

- Introduction
- Gamma decay
- Decay rates
- Selection rules
- Spectroscopic information from gamma decay
- Internal conversion
- Isomers
- Resonance absorption
- Mossbauer effect

Introduction

Most alpha and beta decays (in general most nuclear reactions) leave the final nucleus in an excited state.

These excited states decay rapidly to the ground state through the emission of one or more **Gamma rays**.

Gamma rays are photons of electromagnetic radiation like X rays or visible light.

Gamma rays have energies typically in the range of 0.1 to 10 Mev

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Characteristic of the energy difference between nuclear states
and hence corresponding to wavelengths between 10^4 and 100 fm

Energetic of Gamma decay

Let's consider the decay of a nucleus of mass M at rest, from an initial excited state E_i to a final state E_f

Conservation of total energy: $$E_i = E_f + E_\gamma + T_R$$

Recoil kinetic energy of the nucleus

$$T_R = \frac{p_R^2}{2M}$$

Conservation of momentum: $$0 = p_R + p_\gamma$$

Recoil momentum

nonrelativistic

The nucleus recoils with a momentum equal and opposite to that of the gamma ray.

Using relativictic relationship $E_\gamma = cp_\gamma$ we get: $$\Delta E = E_i - E_f = E_\gamma + \frac{E_\gamma^2}{2Mc^2}$$

Energetic of Gamma decay

The solution is:

$$E_\gamma = Mc^2\left[-1 \pm \left(1 + 2\frac{\Delta E}{Mc^2}\right)^{1/2}\right]$$

Since: ΔE are typically of the order of Mev, while the rest energies Mc^2 are of order of A x 10^3 Mev, A is mass number.

$$\Delta E << Mc^2$$

$$E_\gamma \cong \Delta E - \frac{(\Delta E)^2}{2Mc^2} \quad \Rightarrow \quad E_\gamma \cong \Delta E$$

Some Theory on Electromegnetic radiation

Electromagnetic radiation can be treated either as a classical wave phenomena or as a quantum phenomena

Analyzing radiations from individual atoms and nuclei the quantum description is most appropriate

We can easily understand the quantum calculations of electromagnetic radiation if we first review the classical description

CLASSICAL DESCRIPTION

Static distributions of charges and currents give static electric and magnetic fields.

These fields can be analyzed in terms of the multipole moments of the charge distribution (dipole moment, quadrupole moment, and so on). **See Krane Section 3.5**

If the charge and current distributions vary with time, particularly if they vary sinusoidally with circular frequency omega, then ***a radiation field is produced***

Some Theory on Electromagnetic Radiation

As example, let's consider the lowest multipole order, the dipole filed.

A static ***electric dipole*** consists of equal and opposite charges +q and –q separated by a fixed distance z

The electric dipole moment is: $d = qz$

The static ***magnetic dipole*** can be represented as a circular current loop of a current I enclosing area A.

The magnetic dipole moment is: $\mu = iA$

If the charge oscillate along z axis: $d(t) = qz\cos wt$ ⇒ Production of an electric dipole radiation field

If we vary the current i as: $\mu(t) = iA\cos wt$ ⇒ Production of a magnetic dipole radiation field

Some Theory on Electromagnetic Radiation

Without entering into detailed discussion of electromagnetic theory, we can extend these properties of dipole radiation to multipole radiation in general.

Let's define the index L of the radiation so that 2^L is the multipole order (L=1 for dipole, L=2 for quadrupole, and so on).

Hence, the parity of the radiation field is:

$$\pi(ML) = (-1)^{L+1}$$ For Magnetic field

$$\pi(EL) = (-1)^{L}$$ For Electric field

The radiation power (magnetic (M) or electric (E)) is:

$$P(\sigma L) \frac{2(L+1)c}{\varepsilon_0 L\left[(2L+1)!!\right]^2} (\frac{\omega}{c})^{2L+2} \left[m(\sigma L)\right]^2$$

Some Theory on Electromagnetic Radiation

The general form of the radiation power (magnetic (M) or electric (E)) is:

$$P(\sigma L) = \frac{2(L+1)c}{\varepsilon_0 L\left[(2L+1)!!\right]^2} \left(\frac{\omega}{c}\right)^{2L+2} \left[m(\sigma L)\right]^2$$

where $\sigma = E$ or $\sigma = M$

$m(\sigma L)$ the amplitude of the electric or magnetic multipole moment

The double factorial (2L+1)!! Indicates (2L+1).(2L-1)…..3.1.

Some Theory on Electromagnetic Radiation

QUANTUM MECHANIC DESCRIPTION

To go from classical theory into quantum domain, we must quantize the sources of the radiation field, that is, the classical multipole moments.

Replace multipole moments by multipole operators

Multipole operators change the nucleus from its initial state Ψ_i to the final state Ψ_f

The decay probability is governed by the matrix element of the multipole operator

$$m_{fi}(\sigma L) = \int \Psi_f^* m(\sigma L) \Psi_i dv$$

The integral is carried out over the nucleus volume.

Here we say that the multipole operator $m(\sigma L)$ changes the nuclear state from Ψ_i to Ψ_f while sumultaneously creating a photon of the proper energy, parity and multipole order

Some Theory on Electromagnetic Radiation

Decay constant = probability per unit time for photon emission is:

$$\lambda(\sigma L) = \frac{P(\sigma L)}{\hbar\omega} = \frac{2(L+1)}{\varepsilon_0 \hbar L\left[(2L+1)!!\right]^2} \left(\frac{\omega}{c}\right)^{2L+1} \left[m_{fi}(\sigma L)\right]^2$$

Need to evaluate the matrix elements of $m_{fi}(\sigma L)$, which requires knowledge of the ***initial and final wave functions***

Assumption: The transition is due to a single proton that changes from one shell-model state to another

Then, the ***EL transition probability*** is estimated to be:

$$\lambda(\sigma L) \cong \frac{8\pi(L+1)}{L\left[(2L+1)!!\right]^2} \frac{e^2}{4\pi\varepsilon_0 \hbar c} \left(\frac{E}{\hbar c}\right)^{2L+1} \left[\frac{3}{L+3}\right]^2 cR^{2L}$$

Some Theory on Electromagnetic Radiation

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Then, the ***EL electric transition probability*** is estimated to be:

$$\lambda(EL) \cong \frac{8\pi(L+1)}{L[(2L+1)!!]^2}\frac{e^2}{4\pi\varepsilon_0\hbar c}\left(\frac{E}{\hbar c}\right)^{2L+1}\left[\frac{3}{L+3}\right]^2 cR^{2L}$$

Where R is the nuclear radius $R = R_0 A^{1/3}$

We can make the following estimates for some of the lower multipole oders:

$$\lambda(E1) = 1.0\times 10^{14} A^{2/3} E^3$$

$$\lambda(E2) = 7.3\times 10^{7} A^{4/3} E^5$$

$$\lambda(E3) = 34 A^2 E^7$$

$$\lambda(E4) = 1.1\times 10^{-5} A^{8/3} E^9$$

Weisskopf estimates

Some Theory on Electromagnetic Radiation

Then, the ***ML Magnetic transition probability*** is estimated to be:

$$\lambda(ML) \cong \frac{8\pi(L+1)}{L[(2L+1)!!]^2}\left(\mu_p - \frac{1}{L+1}\right)^2\left(\frac{\hbar}{m_p c}\right)\left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)\left(\frac{E}{\hbar c}\right)^{2L+1}\left[\frac{3}{L+2}\right]^2 cR^{2L-2}$$

Where m_p is the proton mass

We can make the following estimates for some of the lower multipole oders:

$$\lambda(M1) = 5.6\times10^{13} E^3$$

$$\lambda(M2) = 3.5\times10^{7} A^{2/3} E^5$$

$$\lambda(M3) = 16 A^{4/3} E^7$$

$$\lambda(M4) = 4.5\times10^{-6} A^{2} E^9$$

Weisskopf estimates

Some Theory on Electromagnetic Radiation

Based on Weisskopf estimates, two conclusions can be drawn:

1. The lower multipolarities are dominant: increasing the multipole by one unit reduceds the transition probability by a factor of 10-5

2. For a given multipole order, electric radiation is more likely than magnetic radiation by about two orders of magnitude in medium and heavy nuclei.

Angular Momentum and Parity Selection Rules

It is well known that a ***classical electromagnetic field*** produced by oscillating charges and currents transmits not only energy but angular momentum as well.

Also, it is known that the rate at which angular momentum is radiated is proportional to the rate at which energy is radiated

In quantum mechanic limit, the multipole operator of order L includes the factor $Y_{L,M}(\theta,\phi)$ which is associated with an angular momentum L.

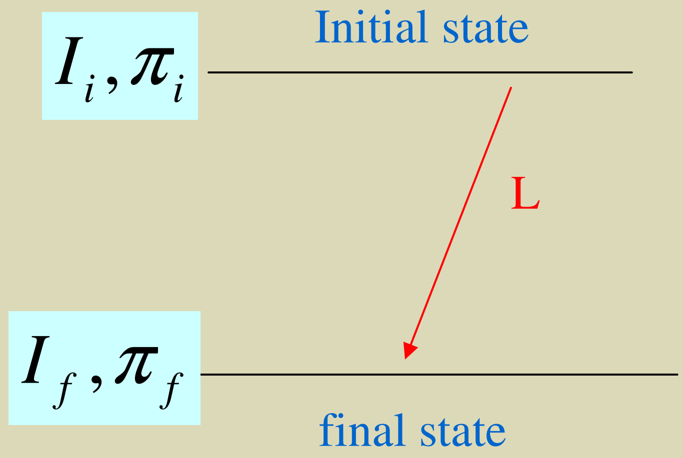
Therefore, a multipole of order L transfers an angular momentum of $L\hbar$ per photon

Consider now, a transition from an initial state of angular momentum I_i and parity π_i to a final state I_f and π_f

Angular Momentum and Parity Selection Rules

Consider now, a transition from an initial state of angular momentum $\boldsymbol{I_i}$ and parity π_i to a final state $\boldsymbol{I_f}$ and π_f

Assume that: $I_i \neq I_f$



Conservation of angular momentum:

$$I_i = L + I_f$$

They form a closed vector triangle:

$$\left| I_i - I_f \right| \leq L \leq I_f + I_i$$

Angular Momentum and Parity Selection Rules

Example: if I_i =3/2 and I_f=5/2 then the radiation field consist of a mixture of dipole (L=1), quadrupole (L=2), octupole (L=3), and hexadecapole (L=4)

TYPE OF RADIATION: ELECTRIC OR MAGNETIC?

Wether the emitted radiation is of the electric or magnetic type is determined by the relative parity of the initial and final levels.

If there is no change in parity → the radiation field must have even parity

If there is a change in parity → the radiation field must have odd parity

However, we have seen that:

$$\pi(ML) = (-1)^{L+1}$$ For Magnetic field

$$\pi(EL) = (-1)^{L}$$ For Electric field

Angular Momentum and Parity Selection Rules

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Taking previous example: Ii =3/2 and If=5/2

If **no change** in parity ⇨ Transition would consist of even electric multipoles (L=2, 4) and odd magnetic multipoles (1, 3):
The radiation field must be: M1, E2, M3, E4

If there is a **change** in parity ⇨ Transition would consist of add electric multipoles (L=1, 3) and even magnetic multipoles (2, 4):
The radiation field must be: E1, M2, E3, M4

Angular Momentum and Parity Selection Rules

In summary the angular and parity selection rules are:

$$\left|I_i - I_f\right| \leq L \leq I_f + I_i$$

No L = 0

$$\Delta\pi = no$$

Even electric, odd magnetic

$$\Delta\pi = yes$$

odd electric, even magnetic

Classically, this corresponds to an electric charge that does not vary in time → no radiation is produced

Exception When $I_i = I_f$ there are no monopole (L=0) transitions

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Hence, the lowest possible gamma multipole order is dipole (L=1)

Angular Momentum and Parity Selection Rules

Special cases:

1. Either I_i or I_f is zero ⟹ Only a pure multipole transition is emitted

2. $I_i = 0$ and $I_f = 0$ ⟹ L = 0 is not permitted for radiative transitions, hence transtion is forbiden by gamma emission.

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These states decay instead through ***internal conversion***

Angular Momentum and Parity Selection Rules

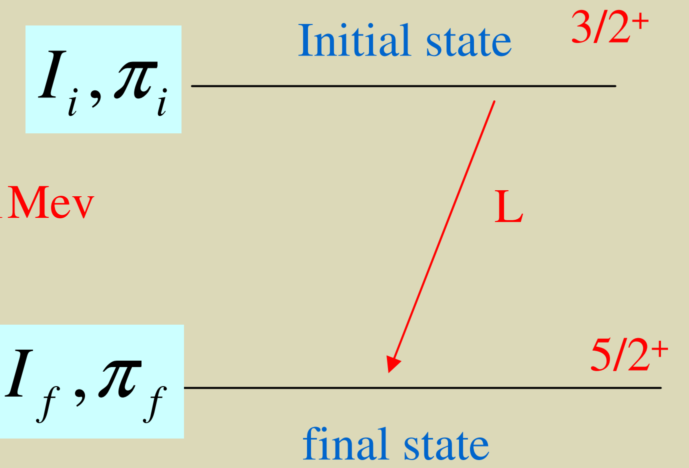
Dominating multipoles:

Usually the spins I_i and I_f have values for which the selection rules permit several miltipoles to be emitted

The single-particle (Weisskopf) estimates permit us to make some general predictions about which multipole is most likely to be emitted

Consider the previous example:

Possible transitions are: M1, E2, M3 and E4



Assume: a nucleus with A=125 and transition energy is E=1Mev

Emission Probabilities are:

$$\lambda(M1) = 1$$

$$\lambda(E2) = 1.4 \times 10^{-3}$$

$$\lambda(M3) = 2.1 \times 10^{-10}$$

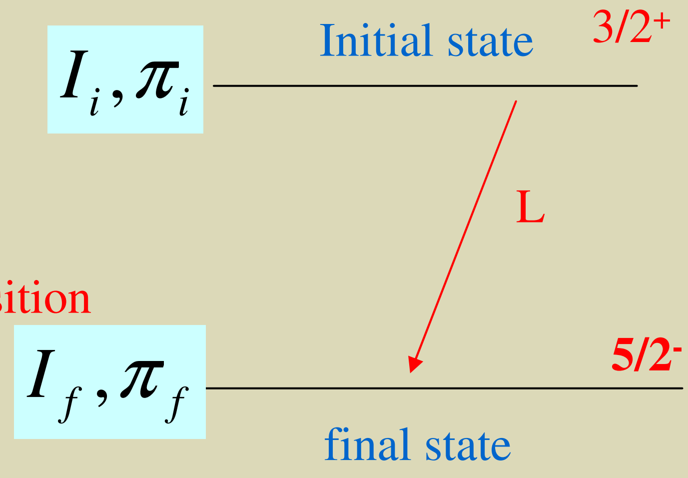
$$\lambda(E4) = 1.3 \times 10^{-13}$$

Multipoles M1 and E2 are more likely to be emitted

We can consider this transition as being composed of M1 radiation with possibly a small mixture of E2

Angular Momentum and Parity Selection Rules

Consider the next case: change in parity, with the same condistion for A and energy E



$$\lambda(E1) = 1$$
$$\lambda(M2) = 2.3\times10^{-7}$$
$$\lambda(E3) = 2.1\times10^{-10}$$
$$\lambda(M4) = 2.1\times10^{-17}$$

⇒ E1 is dominant for this transition

Conclusions: based on the single-particlee estimates

1. Lowest permitted multipole usually dominates.
2. Electric multipole emission is more probable than the same magnetic multipole emission.
3. Emission of multipole L+1 is less probable than emission of multipole L by a factor of the order of about 10^{-5}

Internal Conversion

Internal conversion is an electromagnetic process that competes with gamma emssion.

In this process, the electromagnetic multipole fields of the nucleus do not result in the emission of a photon; instead, the fields interact with the atomic electrons

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This causes one of the electrons to be emitted from the atom

Difference with the Beta decay?

In the internal conversion, the electron is not created in the decay process but rather is a previously existing electron in an atomic orbit.

Important notice:

Internal conversion is ***not a two-step process*** in which a photon is first emitted by the nucleus and then interact with an orbiting electron.

Internal Conversion

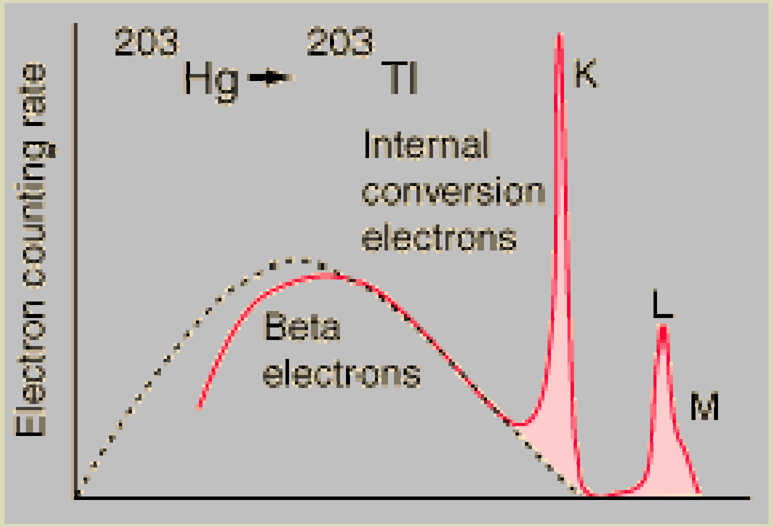
The transition energy ΔE appears as the kinetic energy T_e of the emitted electrons, less the binding energy B that must be supplied to knock the electron out from its atomic shell

$$T_e = \Delta E - B$$

Because the electron binding energy varies with the atomic orbital, for a given transition ΔE there will be internal conversion electrons emitted with different energies.

The observed electron spectrum from a source with a single gamma emission thus consists of a number of individual peaks.

Since most radioactive sources will emit both Beta-decay and internal conversion electrons, We see a curve consisting of peaks riding (due to internal conversion) riding on the continuous Beta spectrum as in the figure



T_e

Internal Conversion

From equation: $$T_e = \Delta E - B$$

It is clear that internal conversion process has a threshold energy equal to the electron binding energy in a particular shell.

As a results, the conversion electrons are labeled according to the electronic shells from which they come: K, L, M, and so on, corresponding to principal atomic quantum numbers n=1, 2, 3,….

Furthermore, if we observe at very high resolution, we can even see the substructure corresponding to the individual electrons in the shell.

EXAMPLE: for L (n=2) shell there are $2s_{1/2}$, $2p_{1/2}$, and $2P_{3/2}$; electrons originating from these shells are called respectively, L_I, L_{II}, and L_{III} conversion electrons

> Conversion process is followed by X-ray emission

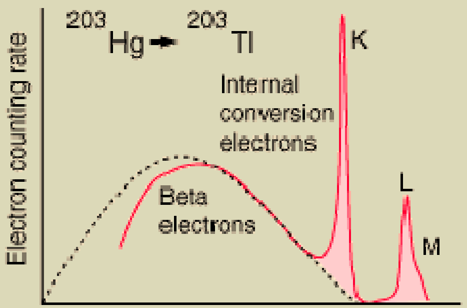
Internal Conversion

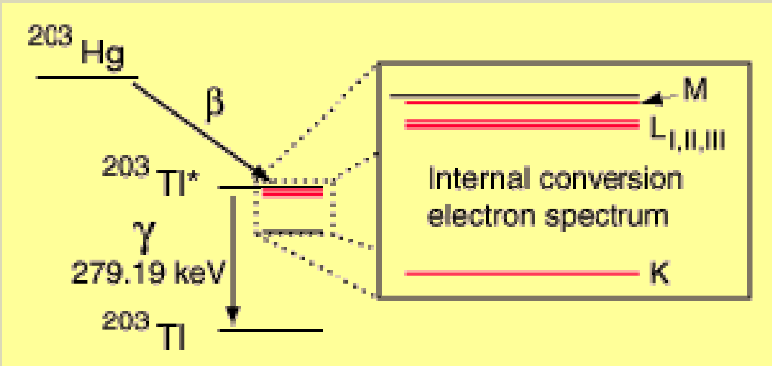
Example:

Consider a Beta decay of ^{203}Hg, which decays to ^{203}Tl by beta emission, leaving the ^{203}Tl in an electromagnetically excited state.

It can proceed to the ground state by emitting a **279.190 keV** gamma ray, or by internal conversion.

The result is a spectrum of internal conversion electrons which will be seen as superimposed upon the electron energy spectrum of the beta emission.

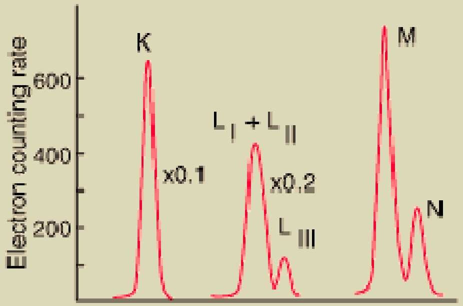




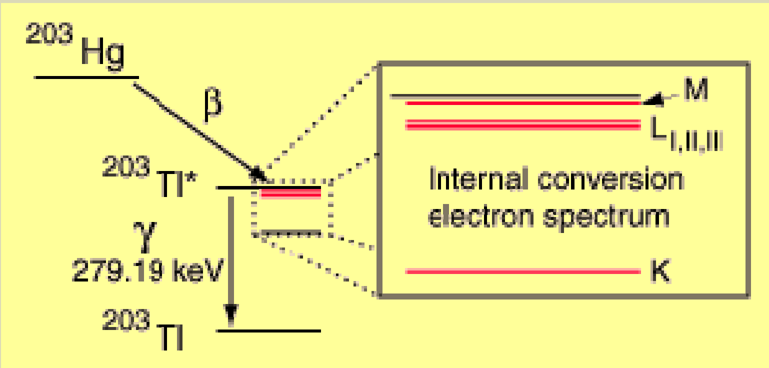
Internal Conversion

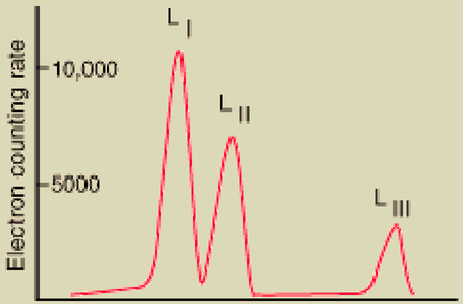
Example:

A close glance into the internal conversion peaks



Higher resolution





Even higher resolution

Internal Conversion

Binding energies for ^{203}Tl		Conversion electron emitted energies
K	85.529 keV	$T_e(K)$= 193.66 kev
L_I	15.347 keV	$T_e(L_I)$=263.84 kev
L_{II}	14.698 keV	$T_e(L_{II})$= 264.49 kev
L_{III}	12.657 keV	$T_e(L_{III})$= 266.53 kev
M	3.704 keV	$T_e(M_I)$=275.49 kev

The energy yield of this electromagnetic transition can be taken as 279.190 keV, so the ejected electrons will have that energy minus their binding energy in the ^{203}Tl daughter atom.

Internal Conversion…some properties

Internal conversion is favoured when the energy gap between nuclear levels is small, and is also the only mode of de-excitation for 0+ → 0+ (i.e. E0) transitions. It is the predominant mode of de-excitation whenever the initial and final spin states are the same, but the multipolarity rules for nonzero initial and final spin states do not necessarily forbid the emission of a gamma ray in such a case.

In some cases, internal conversion is heavily favored over Gamma emission; in others it may be completely negligible compared with Gamma emssion.

As a general rule, it is necessary to correct for internal conversion when calculating the probability for Gamma emission. That is, if we know the half-life of a particular nuclear level, then the total decay probability λ_t arising from internal conversion:

$$\lambda_t = \lambda_\gamma + \lambda_e$$

Internal Conversion

The level decays more rapidly through the combined process than it would if we consider only Gamma emission alone.

Internal conversion coefficient is: $\alpha = \frac{\lambda_e}{\lambda_\gamma}$

Gives the probability of electron emission relative to Gamma emission

Hence total decay probability become:

$$\lambda_t = \lambda_\gamma + \lambda_{e,K} + \lambda_{e,L} + \lambda_{e,M} + ...$$
$$= \lambda_\gamma (1 + \alpha_K + \alpha_L + \alpha_M + ...)$$

Gamma-ray Spectroscopy

The study of the Gamma radiations emitted by radioactive sources is one of the primary means to learn about the structure of the excited nuclear states.

Gamma-ray detection is relatively easy and can be done at high resolution and high precision.

> Knowledge of the locations and properties of the excited states is essential for the evaluation of calculations based on any nuclear model.

> Gamma-ray spectroscopy is the most direct, precise, and often the easiest way to obtain that information

Gamma-ray Spectroscopy

Let's consider how the Gamma-ray experiments might proceed to provide us with the information we need about nuclear excited states:

1. **A spectrum of Gamma rays shows us the energies and intensities of the transitions.**
2. **So-called coincidence measurements give us information about how these transitions might be arranged among the excited states.**
3. **Measuring internal conversion coefficicints can give information about the character of the radiation and the relative spins and parities of the initial and final states.**

See example in Krane: page 351

See Krane also for : Nuclear resonance et Mössbauer effetc

Final exam: 13 June 2007 , from 08 to 11