# Homework 1 on Nuclear Forces

## Exercice 1:

Solve Schrodinger equation for the following well shapes:

1. The 1D infinite well: $$\begin{aligned} V(x) &= \infty \qquad x < 0,\ x > a \\ &= 0 \qquad 0 \le x \le a \end{aligned}$$
2. the infinite spherical well: $$\begin{aligned} V(r) &= \infty \qquad r > a \\ &= 0 \qquad r < a \end{aligned}$$

## Exercise 2:

A simple quantum mechanical model of the deuteron is based on a spherically symmetric potential well given by:

$$\begin{aligned} V(r) &= -V_0 \qquad \text{for } r < b \\ &= 0 \qquad \text{for } r > b \end{aligned}$$

By inserting the Laplacian operator for polar coordinates into the Schrödinger equation:

$$\nabla^2 \psi + \frac{2\mu}{\hbar^2}\left[E - V(r)\right]\psi = 0$$

a. show that the following function is a solution:

$$\psi = \frac{u(r)}{r}$$

where

$$\begin{aligned} u(r) &= A \sin kr \qquad \text{for } r < b \\ u(r) &= B \exp(-ar) \quad \text{for } r > b \end{aligned}$$

Include in your answer explicit expressions for **k** and **a**.

b. Use the continuity and normalisation conditions to find the relationship between $\mathbf{V_0}$ and **b** Given that the binding energy of the deuteron is **-2.2 MeV** and assuming $\mathbf{V_0}$ is **35 MeV**, estimate the value of **b**.

c. For a spherically symmetric potential well of the above given radius what would be the approximate depth if the binding energy of the deuteron were **-10 MeV**? Estimate the value of the magnitude of $\mathbf{V_0}$ below which no bound state will exist.

d. Using the values $\boldsymbol{\mu_{d(L=0)}} = \mathbf{0.8798}\ \boldsymbol{\mu_N}$ and $\boldsymbol{\mu_{d(L=2)}} = \mathbf{0.3101}\ \boldsymbol{\mu_N}$, find the percentage admixture of the (***L*= 2**) state if the deuteron magnetic momemt were given by $\boldsymbol{\mu_d} = \mathbf{0.8325}\ \boldsymbol{\mu_N}$. Note that, as indicated in the lecture, $\mathbf{a_0^2 + a_2^2 = 1}$.

**Exercise 3:**

The measured spin of deuteron is found to be $I=1$ . since the neutron and proton spins can be either parallel or antiparallel, show that there are four possible ways, presented in the lecture, to couple $\mathbf{S_n}$, $\mathbf{S_p}$, and $\ell$ to get a total $I=1$.

**Exercise 4:**

Show that in a nucleon-nucleon scattering problem, if the incident energy is far below 20 Mev, then the assumption that the relative orbital angular momentum is equal to zero ($\ell = 0$) is justified.

**Exercice 5:**

a. What is the effect of parity and time reversal operators on individual spins, $\vec{S}_1$, $\vec{S}_2$.

b. Show that under parity and time reversal the terms such as : $\vec{S}_1$, $\vec{S}_2$, and $A\vec{S}_1 + B\vec{S}_2$ are excluded from being in the spin-dependent potential term.

c. Show that terms like: $\vec{S}_1^{\,2}$, $\vec{S}_2^{\,2}$, and $\vec{S}_1 \bullet \vec{S}_2$ are allowed in the spin-dependent potential term.

d. Show that the term $V_{so}(r)\vec{\ell} \bullet \vec{s}$ fulfils the symmetry requirement (parity and time reversal)