PHYS580 Final Exam

Exercise 1:

1. Give two independent arguments that prove the strong dependence of the nuclear force from relative spins of nucleons.
2. Give a brief description on the fundamental physical mechanism behind the nucleon-nucleon force?
3. What are the evidences of the existence of the shell model in the nuclear structure?
4. Give the reasons behind the necessity to consider a second particle (neutrino) that must be emitted in the beta decay.
5. What is the fundamental difference between the internal capture and the beta decay.

Exercise 2:

Taking into account that experiments indicated that nuclear forces are invariant with respect to parity and time-reversal:

a. Show that under parity and time reversal the terms such as : $\vec{S}_1$, $\vec{S}_2$, and $A\vec{S}_1 + B\vec{S}_2$ are excluded from being in the spin-dependent potential term.

b. Show that terms like: $\vec{S}_1^{\,2}$, $\vec{S}_2^{\,2}$, and $\vec{S}_1 \bullet \vec{S}_2$ are allowed in the spin-dependent potential term.

c. Show that the term $V_{so}(r)\vec{\ell} \bullet \vec{s}$ fulfils the symmetry requirement (parity and time reversal)

Exercise 3:

Find the configuration of the protons and neutrons in the incomplete shells and hence the ground state **spin** and **parity** assignments for the following nuclei

$$^{7}\mathrm{Li}_{3} \; ; \; ^{23}\mathrm{Na}_{11} \; ; \; ^{33}\mathrm{S}_{16} \; ; \; ^{41}\mathrm{Sc}_{21}$$

under the assumption that the ordering of the lowest single particle nuclear energy levels is

$$1s_{1/2} \; ; \; 1p_{3/2} \; ; \; 1p_{1/2} \; ; \; 1d_{5/2} \; ; \; 2s_{1/2} \; ; \; 1d_{3/2} \; ; \; 1f_{7/2} \; ; \; 2p_{3/2}.$$

In this model the first excited states can be produced either

1. by excitation of the unpaired nucleon into the next **higher** subshell, or

2. by pairing this nucleon with another excited from the next **lower** subshell.

Determine the spin and parity for these two types of excited state for each of the four given nuclides.

Exercise 4:

Consider the α decay of ^{242}Cm from an initial state, with spin zero and parity even, to many different states (ground and excited states) of the daughter nucleus of ^{238}Pu. The ^{238}Pu nucleus has states with the following spin and parity information: 0^+ (ground state), 1^-, 2^+, 2^-, 3^+, 3^-, 4^+, 4^-, 5^+, 5^-, 6^+, 6^-. From these states, find the α forbidden transitions?

Exercise 5:

The isotope $^{14}O_8$ is a positron emitter, decaying to an excited state of $^{14}N_7$. The gamma rays from this latter have an energy of **2.313 MeV** and the maximum energy of the positrons is **1.835 MeV**. The mass of $^{14}N_7$ is **14.003074 u** and that of the electron is **0.000549 u**. Write the equation for the decay of the oxygen isotope and sketch an energy level diagram for the process. Given that one unified mass unit (**u**) is equal to **931.502 MeV/c^2** find the mass of $^{14}O_8$.

Problem:

Consider a one-dimensional rectangular potential barrier V(x):

$$V(x) = \begin{cases} 0 & x\langle 0,\ x\rangle a \\ V_0 & 0\langle x \langle a \end{cases}$$ where V_0 is positive

A particle with energy E < V_0 approaches the barrier from the left (see figure1).

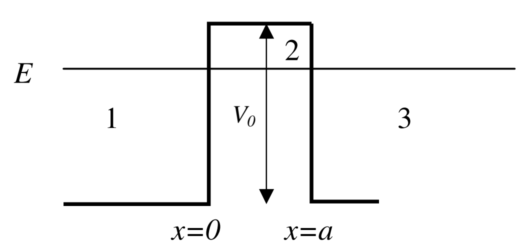

Figure 1

1) Find the wave function in each region.
2) By definition, the transmission coefficient T is defined as the transmitted current divided by the incident current:

$T = \frac{j_{transmitted}}{j_{incident}}$ where $j = \frac{\hbar}{2mi}(\psi^* \cdot \frac{\partial \psi}{\partial x} - \psi \cdot \frac{\partial \psi^*}{\partial x})$

evaluate T in terms of the amplitudes of transmitted and incident wave functions.

3) Using the boundary conditions prove that: $T = \frac{1}{1 + \frac{1}{4}\frac{V_0^2}{E(V_0 - E)}\sinh^2 k_2 a}$ where k_2 is the wave vector in region 2.

The theory behind the α decay is a quantum mechanical-tunnelling problem.
Figure 2 shows a plot of the potential energy between the α particle and the residual nucleus for various distances between their centres. According to α decay theory, the potential comprises the nuclear well and the Coulomb barrier preventing the α to escape easily.

The disintegration constant of an α emitter is given in one-body theory by:
$\lambda = f \cdot P$, where *f* is the frequency with which the α particle presents itself at the barrier and P is the probability of the transmission through the barrier. It must be obtained from a quantum mechanical calculation similar to the one-dimensional problem above.

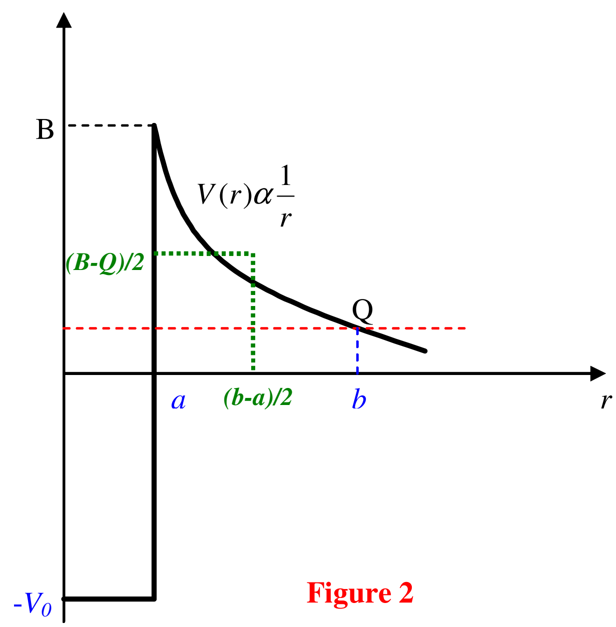

Figure 2

The horizontal line Q is the disintegration energy.

The calculation of the barrier penetration for the three-dimensional Coulomb potential is rather complicated. The coulomb barrier has height of B at $r = a$.

1) Give the relationship between B and a?

However, since we are interested only in order-of-magnitude estimates, we ignore the angular dependence of the Schrödinger equation and consider the potential as effectively one-dimensional. Furthermore, we can replace the Coulomb potential by the barrier presented by green dashed lines. Hence, we can take a representative average height for the coulomb potential to be $\frac{1}{2}(B-Q)$. Similarly, we choose a representative average width to be $\frac{1}{2}(b-a)$.

For a typical heavy nucleus with *Z=90* and *a=7.5 fm*, the barrier height is about 34 Mev and *Q=6 Mev.*

2) Calculate the value of the wave vector in region 2 (k_2).
3) Calculate the radius b at which the α particle "leaves" the barrier?
4) Noticing that $k_2 \cdot \frac{1}{2}(b-a) >> 1$, show that the transmission probability (found in Problem1) can be approximated to: $T = P \cong \exp(-2k_2 \cdot (1/2) \cdot (b-a))$
5) Calculate in this case the values of *P*, λ and $t_{1/2}$ if the frequency $f = 6 \cdot 10^{21} / s$.
6) By introducing a slight change to Q that become 5Mev, recalculate the half life?? What do you observe?

The exact quantum mechanical calculation is very similar in spirit to the crude estimate above. We can think of the Coulomb barrier as made up of a sequence of infinitesimal rectangular barriers of height $V(r) = zZ'e^2 / 4\pi\varepsilon_0 r$ and width *dr*. where ze is the charge of α particle and $Z'e$ is the charge of the daughter nucleus.

7) Write the expression for the probability to penetrate each infinitesimal barrier, which extend from *r* to *r+dr.*
8) prove that the total probability to penetrate the complete barrier is: $P = \exp(-2 \cdot G)$, where G is the so-called Garow factor:

$$G = \sqrt{\frac{2m}{\hbar^2}} \int_a^b [V(r) - Q]^{1/2} dr$$

Constants

Charge of the electron **e = 1.6 10^{-19} C**
Mass of **He**: $m_{He} = 4.002603u$
Planck constant **h = 6.626 10^{-34} J s = 4.136 10^{-15} eV s**
Speed of light in free space **c = 3.00 10^{8} m s^{-1}**
Permittivity of free space **ε_0 = 8.85 10^{-12} F m^{-1}**
$\hbar c = 197.33$ Mev.fm
Unified atomic mass unit **1u = 1.66 10^{-27} kg = 931.502 MeV/c^{2}**

بالتوفيق