

## *Phys487 Final Exam*

### ***Problem 1: Comprehension***

- a. Why light nuclei are better moderators than heavy nuclei.
- b. What effect does the presence of the fuel have on the diffusion of neutrons (scattering and absorption).  
the fuel does not affect the scattering properties, but has a marked effect on its absorption properties.
- c. What is the physical interpretation of the mean free path of a neutron when interacting with matter.

➤ **HENCE:** Average path length between successive collisions for neutrons moving in a medium of macroscopic cross section  $\Sigma$  is equal to  $\frac{1}{\Sigma}$

- d. Explain the role of the reflector in a reactor core?  
The effect of neutron leakage can be reduced by the employment of a neutron **reflector** which surrounds the active core of the reactor and which reflects or scatters the escaped neutrons back into the core.
- e. What is the cause of the neutron diffusion in a reactor assembly?  
Hence, diffusion of neutrons is a consequence of the **non-uniform neutron density** in the reactor assembly
- f. What is the physical meaning of the thermal diffusion length and fast diffusion length?

#### **Thermal diffusion length**

This length  $L$  is a measure of the air-line distance a neutron travels between the point of its origin as a thermal neutron and the point of its absorption

#### **Fast diffusion length**

It is a measure of the distance a fission neutron has traveled away from its point of creation by the time it reaches thermal energies

g. On which basis the reactors are classified?

**Answer:** Reactors are classified based on a variety of characteristic features :

- 1) Type of fuel used
- 2) Average neutron energy at which the greater part of all fissions occur
- 3) Moderator materials used
- 4) Arrangement and spatial disposition of fuel and moderator
- 5) Purpose of the reactor

h. Using a simple useful equation, demonstrate that the energy transfer between a neutron and a heavy nuclei is almost zero?

For  $A > 10$ , an approximation good to within 1% is:  $\xi = \frac{2}{A + \frac{2}{3}}$

For large  $A$ ,  $\xi \rightarrow 0$

**Problem 2: Chapter 4, Problem 1**

An indium foil of  $2 \text{ cm}^2$  cross section and  $10^{-3} \text{ cm}$  thickness is exposed to a broad beam of neutrons of uniform energy. If the neutron flux is  $5 \times 10^9 \text{ neutrons/cm}^2 \text{ sec}$  and the microscopic absorption cross section for these neutrons is  $190 \text{ barns}$ , calculate the number of neutron captures that will occur during a 3 min exposure of the foil. For  $^{115}\text{In}$ ,  $\rho = 7.29 \text{ gr/cm}^3$ .

**Reaction rate  $\rightarrow$  number of reactions per second:**  $r = N_0 \cdot t \cdot \sigma \cdot I \cdot A$

**Number of nuclei per  $\text{cm}^3$ :**  $N_0 = \frac{N_{av} \cdot \rho}{M} = 3.8 \times 10^{22} \text{ nuclei/cm}^3$

$r = 3.8 \times 10^{22} \times 10^{-3} \times 190 \times 10^{-24} \times 5 \times 10^9 \times 2 = 7.22 \times 10^7 \text{ reaction/sec}$

**Number of reactions (captures) in 3 min:**  $r \times 3 \times 60 = 1.3 \times 10^{10} \text{ reactions}$

**Problem 3: Chapter 6, problem 6**

It is desired to reduce the intensity of a thermal neutron beam by a factor of 1000 by interposing a cadmium foil with a thickness of 0.061 cm. Calculate the microscopic absorption cross section for cadmium. For Cadmium:  $\rho = 8.65 \text{ gr/cm}^3$ ,  $M = 113 \text{ gr}$ .

$$I = I_0 e^{-\Sigma_a X}, \Sigma_a = N_0 \sigma_a \Rightarrow \sigma_a = 2450 \text{ barns}$$

**Problem 4: Chapter7, Problem4, page 241**

Starting a neutron cycle with 1000 fast fission neutrons (first generation). Calculate:

- a) the number of neutrons produced by fast fissions

$$n_0 \varepsilon = 1000 \times 1.029 = 1029$$

- b) the number of neutrons absorbed in uranium fuel before reaching thermal energies

$$n_0 \varepsilon (1 - p) = 1029 \times (1 - 0.889) = 109$$

- c) the number of neutrons reaching thermal energies

$$n_0 \varepsilon p l_f = 1029 \times 0.889 \times 0.956 = 875$$

- d) the number of thermal neutrons absorbed in fuel

$$n_0 \varepsilon p l_f l_{th} f = 1029 \times 0.889 \times 0.956 = 875 \times 0.945 \times 0.910 = 752$$

- e) the number of neutrons escaping through fast leakage

$$n_0 \varepsilon (1 - l_f) = 1029 \times (1 - 0.91) = 45$$

- f) the number of neutrons escaping through thermal leakage

$$n_0 \varepsilon p l_f (1 - l_{th}) = 875 \times (1 - 0.945) = 48$$

- g) the number of fast fission neutrons of the second generation

$$n_0 k_{eff} = n_0 \varepsilon p f \eta l_f l_{th} = 752 \times 1.34 = 1008$$

$$\varepsilon = 1.029, p = 0.889, f = 0.910, l_f = 0.956, l_{th} = 0.945, \eta = 1.34$$

**Problem 5: Chapter 8, Example 8.3 and 8.6, page 252**

1) Calculate the diffusion length for thermal neutrons in graphite used as a moderator, if

$$\sigma_a = 3.2 \text{ mb}; \sigma_s = 4.8 \text{ barns}; \rho_{\text{graphite}} = 1.62 \text{ gr/cm}^3$$

$$\Sigma_a = N_0 \sigma_a = 2.61 \times 10^{-4} \text{ cm}^{-1}, \lambda_a = 3840 \text{ cm}$$

$$\Sigma_s = N_0 \sigma_s = 0.415 \text{ cm}^{-1}, \lambda_s = 2.40 \text{ cm}$$

$$\lambda_{tr} = \frac{\lambda_s}{1 - (2/3A)} = 2.54 \text{ cm}$$

$$L^2 = \frac{\lambda_{tr} \lambda_a}{3} \Rightarrow L = 57.2 \text{ cm}$$

2) How much the diffusion length become when we consider a homogeneous mixture of 1 atom of  $U^{235}$  per  $10^4$  atoms of graphite, if  $\sigma_a^{U^{235}} = 698 \text{ barns}$

$$f = \frac{1}{1 + \frac{\sum_{am}}{\sum_{a(235)}}} = 0.958$$

$$L^2 = L_m^2 (1 - f) = 57.2^2 \times 0.042 = 2.4 \Rightarrow L = 11.72 \text{ cm}$$

3) Compare and discuss the results found in a) and b).

The diffusion length with fuel is less than without fuel. This is logic since introducing fuel affect the probability of absorption of neutrons. This means that neutrons will travel less distance before be absorbed by the fuel.

**Problem 6: Examples 9.1 and 9.7, page 274**

Consider a critical thermal cubic reactor employing  $U^{235}$  and graphite in an atom ratio of 1: 100000.

1) Calculate the total leakage (fast and thermal). **Example 9.1, page 274**

$$f = \frac{1}{1 + \frac{N_{0(C)} \sigma_{a(c)}}{N_{0(235)} \sigma_{a(235)}}} = 0.699$$

$$k_{\infty} = 2.08 \times 0.699 = 1.45$$

$$\text{Nonleakage factor: } l_{th} l_f = \frac{k_{eff}}{k_{\infty}} = \frac{1}{1.45} = 0.69$$

$$\text{Leakage: } 1 - 0.69 = 0.31$$

2) Calculate the extrapolation length (distance).

$$d = 0.71 \cdot \lambda_{tr} = 0.71 \cdot \frac{\lambda_s}{1 - \frac{2}{3A}} = 2 \text{ cm}$$

- 3) Calculate the critical size if  $B = 0.018$ . **Example 9.7, page 289**

$$a = 3^{1/2} \frac{\pi}{B} = 300 \text{ cm},$$

$$a_m = 300 - 2 \times d = 296 \text{ cm}$$

- 4) Calculate the critical mass of  $U^{235}$ .

$$\frac{M_u}{M_m} = \frac{N_u}{N_m} \times \frac{235}{12}, \text{ with } \frac{N_u}{N_m} = 10^{-5}$$

$$\text{and } \frac{M_u}{\rho_u} + \frac{M_m}{\rho_m} = V = a_m^3$$

$$M_u = 8.25 \text{ kg}$$

Graphite:  $\sigma_a = 0.003 \text{ barn}$ ;  $L_m = 54 \text{ cm}$ ;  $\tau_0 = 364 \text{ cm}^2$ ;  $\lambda_s = 2.60 \text{ cm}$ ;  $\rho_m = 1.62 \text{ gr/cm}^3$

Uranium:  $\eta = 2.08$ ;  $p = 1$ ;  $\epsilon = 1$ ;  $\rho_u = 18.7 \text{ gr/cm}^3$ ,  $\sigma_a^{U^{235}} = 698 \text{ barns}$

Some useful equations and quantity values

$$\bar{E} = \frac{3}{2}kT \quad (\bar{v^2})^{1/2} = v_{rms} = \left(\frac{3kT}{m}\right)^{1/2}$$

$$\Delta T = T_n - T = 0.89T \cdot A \cdot \left(\frac{\sum a}{\sum s}\right)$$

$$\frac{E_1}{E_0} = \frac{v_1^2}{v_c^2} = \frac{1 + A^2 + 2A \cos \phi}{(1 + A)^2}$$

$$\frac{(A-1)^2}{(1+A)^2} = \alpha$$

$$\overline{\left(\frac{\Delta E}{E}\right)} = \left(\frac{E - E_0}{E}\right) = \left(\frac{1 - \alpha}{2}\right)$$

$$\xi = \log\left(\frac{E_0}{E}\right) = \frac{\int_E^{E_0} \log\left(\frac{E_0}{E}\right) P(E) dE}{\int_E^{E_0} P(E) dE}$$

$$sdp = \frac{\xi}{\lambda_s}$$

$$d = 0.71\lambda_{tr}$$

$$\lambda_{tr} = \frac{\lambda_s}{1 - (2/3A)}$$

$$L^2 = \frac{\lambda_r \lambda_a}{3}$$

$$f = \frac{1}{1 + \frac{\sum_{am}}{\sum_{a0}}}$$

The geometrical buckling for a parallelepiped of sides  $a$ ,  $b$  and  $c$  is given by:

$$B^2 = \frac{\pi^2}{a^2} + \frac{\pi^2}{b^2} + \frac{\pi^2}{c^2}$$