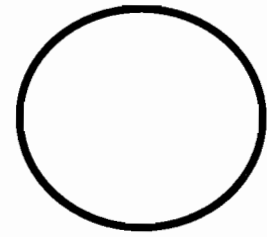


M -- 105



MAX.. MARKS: 100

TIME : 3 Hours

DEPARTMENT OF MATHEMATICS
KING SAUD UNIVERSITY

Final Examination Semester I (1426/1427)

Number of Pages:7.

NAME : _____

REG.NO: _____

GROUP NO: _____

NAME OF TEACHER: _____

Question No	Marks
1 - 25	
26,27	
28	
29,30	
31,32	
Total Marks	

Mark {a,b,c,or d} for the correct answer in the space below for Q.1. - Q.25

Q.1. The solution set of the inequality $20 - 3x > 5$ is
 (a) $(-\infty, -5)$, (b) $(5, +\infty)$, (c) $(-\infty, 5)$, (d) None of these.

Q.2. The domain of the function $f(x) = \frac{5x}{\sqrt{25-x^2}}$, is
 (a) $[-5, 5]$, (b) $(-5, 5)$, (c) $(5, +\infty)$, (d) None of these.

Q.3. If $f(x) = x^{-2}$ and $g(x) = x^2$, then $g \circ f(x)$ is
 (a) x^{-4} , (b) 1, (c) x^4 , (d) None of these.

Q.4. $f(x) = \frac{1}{1-|x|}$ is continuous at
 (a) 2, -1, (b) -1, 0, (c) -1, 1, (d) None of these.

Q.5. $f(x) = (1-x^2)^\alpha$ is continuous at $x = 1$ and $x = -1$, if
 (a) $\alpha = -1$, (b) $\alpha = 1$, (c) $\alpha = -2$, (d) None of these.

Q.6. The vertical asymptote to the graph of $f(x) = \frac{x}{3x+x^3}$ is
 (a) -3, 0 (b) -3, (c) 0, (d) None of these.

Q.7. The horizontal asymptote to the graph of $f(x) = \frac{3-x^2}{3+x^2}$ is
 (a) $y = -1$, (b) $x = 1$, (c) $y = 1$, (d) None of these.

Q.8. If $\lim_{x \rightarrow a} \frac{\sin(\pi-x)}{\pi-x} = 1$, then 'a' is equal to
 (a) 0, (b) π , (c) $-\pi$, (d) None of these.

Q.9. If $\frac{x^2-4}{x-2} \leq f(x) \leq \frac{x+\sin x}{x}$, then $\lim_{x \rightarrow 0} f(x)$ is equal to
 (a) 1, (b) 2, (c) not defined, (d) None of these.

Q.10. The liner approximation of the $f(x) = (1+x)^3$ near $a = 0$ is
 (a) $1+x$, (b) $1+3x$, (c) $(1+x)^2$, (d) none of these

Q.11. The vertical tangent line of a function $f(x) = 2 + 3x^{\frac{2}{3}}$ is
 (a) $y = 0$ (b) $x = 0$ (c) $x = 2$, (d) None of these.

Q.12. If $y = \sin \sqrt{x}$, then $\frac{dy}{dx}$ is
 (a) $\frac{1}{2} \cos \sqrt{x}$, (b) $2\sqrt{x} \cos \sqrt{x}$, (c) $\frac{\cos \sqrt{x}}{2\sqrt{x}}$, (d) None of these

Q.13. Rate of increase of $A = \frac{x^2}{2}$, when width x increases at the rate 4 cm/sec,
 and its length is 5 cm is
 (a) $200 \text{ cm}^2/\text{sec}$, (b) $10 \text{ cm}^2/\text{sec}$, (c) $20 \text{ cm}^2/\text{sec}$, (d) None of these.

Q.No.	Mark a,b,c, d
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Total	

Q.14. Slope of tangent line to the graph of $xy^2 = 1$, at point (1, 1) is

- (a) -2, (b) $-\frac{1}{2}$, (c) $\frac{1}{2}$, (d) None of these.

Q.15. If $f(x) = (x-1)(x^2 - x - 2)$ satisfies Rolle's theorem on $[-1, k]$, then value of k is

- (a) -2, (b) 2, (c) 3, (d) None of these.

Q.16. If $f(x) = x(x-k)^2$ has point of inflection at $x = 1$, then k equals

- (a) 3, (b) 1, (c) 2, (d) None of these.

Q.17. If $f(x) = x^5 - 80x^2$, it is concave upwards in

- (a) $(-\infty, 2)$, (b) $(2, \infty)$, (c) $(-\infty, \infty)$, (d) None of these.

Q.18. Critical number for $y = 2 \sin x + x$ on $[0, \pi]$ is

- (a) $\frac{\pi}{2}$, (b) $\frac{2\pi}{3}$, (c) $\frac{\pi}{3}$, (d) None of these.

Q.19. If $f(x) = \tan^{-1}(x^2 + 1)$, then $f'(1)$

- (a) $\frac{2}{5}$, (b) $\frac{4}{5}$, (c) $\frac{3}{5}$, (d) None of these.

Q.20. If $f(x) = \sqrt{x^2 + 3}$ on $[0, \infty)$, then its inverse function is

- (a) $g(x) = x^2 - 3$, (b) $g(x) = \sqrt{x^2 - 3}$, (c) $g(x) = x^2 + 3$, (d) None of these.

Q.21. The domain of the function $f(x) = \cos^{-1}(2x + 3)$, is

- (a) $[-2, -1]$, (b) $[-1, 1]$, (c) $[0, \pi]$, (d) None of these.

Q.22. The exact value of $\cos\left(\sin^{-1}\left(\frac{1}{3}\right)\right)$ equals

- (a) $\frac{3}{4}$, (b) $\frac{4}{5}$, (c) $\frac{5}{4}$, (d) None of these.

Q.23. The length of the major axis of the ellipse $9x^2 + 4y^2 = 36$ is

- (a) 9, (b) 4, (c) 3, (d) None of these.

Q.24. Asymptotes of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ are

- (a) $y = \pm \frac{3}{2}x$, (b) $y = \pm \frac{2}{3}x$, (c) $y = \pm x$, (d) None of these.

Q.25. The parabola $y^2 = -8x$ has its focus at the point

- (a) $(0, -2)$, (b) $(0, 0)$, (c) $(-2, 0)$, (d) None of these.

Question.26. Find the horizontal and vertical asymptotes for $f(x) = \frac{x+1-\sqrt{x}}{x^2-2x+1}$. [8]

Question.27. Suppose a spherical snowball is melting and its radius is decreasing at a constant rate **1.5 ft/min**. How fast is the volume changing when the radius is **6 inches**. [6]

Question.28. Let $f(x) = x^{\frac{1}{3}}$, determine a vertical tangent line, find intervals of increasing and decreasing of the function, discuss concavity, find point of inflection, local extrema and sketch the graph. [12]

Question.29. Find derivative of the function $f(x) = \sin^{-1}(\sqrt{1-x^2}) + \tan(2x)$ [4]

Question.30. Prove that the function $f(x) = 5x^{\frac{2}{3}} - x^{\frac{5}{3}}$ satisfies the hypotheses of Rolle's theorem on the interval $[0,5]$ and find all numbers c satisfying the conclusion of Rolle's theorem . [8]

Question.31. Use implicit differentiation to find $\frac{dy}{dx}$ if y satisfies the equation

$$x \cos^{-1} y + x^2 y + y^3 = 4$$

[6]

Question.32. Discuss and sketch the graph of the conic

$$x^2 - 2y^2 - 4x + 12y + 2 = 0.$$

[6]