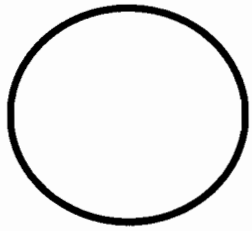
M - 105



MAX. MARKS: 100

TIME : 3 hours
Number of pages:7.

DEPARTMENT OF MATHEMATICS
KING SAUD UNIVERSITY
Final Examination Semester II (1425/1426)

Question No	Marks
1 - 25	
26	
27	
28	
29	
Total Marks	

NAME :_____

REG.NO:_____

GROUP NO:_____

NAME OF TEACHER:_____

Mark {a,b,c,or d} for the correct answer in the space below for Q.1. - Q.25

Marks 2 x 25 = 50

Q.1. The solution set of the inequality $\frac{6}{x^2+6} > 0$ is

(a) $(-\infty,+\infty)$, (b) $(0,+\infty)$, (c) $(-1,1)$, (d) None of these.

Q.2. The domain of the function $f(x)=\frac{\sqrt{25-x^2}}{x}$ is

(a) $(-\infty,+\infty)-\{0\}$, (b) $(-\infty,-5)$, (c) $(-\infty,5)$, (d) None of these.

Q.3. If $f(x)=\frac{1}{x^{-2}}, g(x)=\frac{1}{x^2}$, then $fog(x)$ is

(a) x^{-2}, (b) 1, (c) x^{-4}, (d) None of these.

Q.4. If $\lim_{x\to-\infty}\frac{1+x^{\frac{1}{3}}}{1-x^{\frac{1}{3}}}=k$, then the value of k is

(a) -1, (b) 0, (c) ∞, (d) None of these.

Q.5. The vertical asymptote to the graph of $f(x)=\frac{x^3+x}{x-x^2}$ is

(a) y = -1, (b) $x=\pm1$, (c) x = -1, (d) None of these.

Q.6. The horizontal asymptote to the graph of $f(x)=\frac{8+x^2}{x^2+x}$ is

(a) y = 1, (b) x =1, (c) y = 2, (d) None of these.

Q.7. If $\lim_{x\to0}\frac{\tan(kx)}{x}=\frac{2}{3}$, then k is

(a) $\frac{2}{3}$, (b) 1, (c) $\frac{3}{2}$, (d) None of these.

Q.8. $\lim_{x\to0}\frac{\sin(2x)+\tan x}{x}$ is equal to

(a)2 (b) 3, (c) 1, (d) None of these.

Q.9. If $\frac{x^2-4}{x-2}\le f(x)\le\frac{x+\sin x}{x}$, then $\lim_{x\to0} f(x)$ is equal to

(a) 1, (b) 2, (c) not defined, (d) None of these.

Q.10. The vertical tangent line to the graph of the $f(x)=2\sqrt{1+x}$ is

(a) $y=x+1$, (b) $x=0$, (c) $x=-1$, (d) None of these.

Q.11. The function $f(x)=|1-x|$ is not differentiable at

(a) x = 1, (b) x = -1, (c) $x=0$, (d) None of these.

Q.12. The function $f(x)=\frac{\sqrt{x-1}}{x+4}$ is continuous on

(a) $(-4,+\infty)$, (b) $(-1,4)$, (c) $(-\infty,-4)$, (d) None of these

Q.13. The slope of tangent line to the graph of $y=\frac{4}{1-x}$, at $x=2$ is

(a) 0, (b) 4 (c) -4, (d) None of these.

Q.No.	Mark a,b,c, d
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Q.14. The function $f(x) = kx + \frac{1}{x}$ satisfies Rolle's theorem on $\left[\frac{1}{2}, 2\right]$, then the value of k must be

(a) -1, (b) 0 , (c) 1, (d) None of these.

Q.15. If $x^2 + y^3 = 5$ and $\frac{dx}{dt} = 3$, then the value of $\frac{dy}{dt}$ at point (2 , 1) is

(a) - 4, (b) 0, (c) 4, (d) None of these.

Q.16. Critical point for $y = \frac{1}{2}\sin^2 x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

(a) $x = 0$, (b) $x = \frac{2\pi}{3}$, (c) $x = \frac{\pi}{3}$, (d) None of these.

Q.17. Suppose f is a differentiable function with only one critical point $x = 1$, if $f''(1) = 4$ then at $x = 1$, $f(x)$ has

(a) local max., (b) local min., (c) point of inflection, (d) None of these.

Q.18. $f(x) = 2x^2 - 8x + 9$ has local extrema at

(a) $x = -2$. (b) $x = 9$, (c) $x = 2$, (d) None of these.

Q.19. $f(x) = 2x^2 + \frac{2}{x}$, then point of inflection is at

(a) 1, (b) 0, (c) -1, (d) None of these.

Q.20. If $x^2y^2 = 1$, then y' is

(a) $\frac{y}{x}$, (b) $-\frac{y}{x}$, (c) $\frac{x}{y}$, (d) None of these.

Q.21. If $f(x) = \tan^{-1}\sqrt{x}$, then $f'(1)$ is

(a) 4, (b) $\frac{1}{4}$, (c) $-\frac{1}{4}$, (d) None of these.

Q.22. Inverse function of $y = \sqrt{x-3}$, $x \geq 3$ is

(a) $y = x^2 + 3$, (b) $y = \sqrt{x+3}$, (c) $x = \sqrt{y-3}$, (d) None of these.

Q.23. The vertex of the parabola $x^2 + 4x - 2 + y = 0$ is

(a) (0,-2) . (b) (-2,6). (c) (2,1). (d) None of these

Q.24. Asymptotes of the hyperbola $25y^2 = 4x^2 + 100$ are

(a) $y = \pm\frac{4}{25}x$, (b) $y = \pm\frac{5}{4}x$, (c) $y = \pm\frac{2}{5}x$, (d) None of these.

Q.25. The length of the minor axis of the ellipse $9y^2 + 4x^2 = 36$ is

(a) 3, (b) 6, (c) 4, (d) None of these.

Q.26.(a) Solve the inequality **[6]**

$$\frac{x-1}{3} \geq 2 - \frac{2}{x}$$

Q.26.(b) Let $f(x) = 2x^2 - 1$, and $g(x) = 1 - kx$, find values of k such that **[5]**

$$(fog)(1) = (gof)(1)$$

Q.27.(a) Find the limit if it exists **[5]**

$$\lim_{x \to 0} \frac{\sin(x^2) + 3x(1 - \cos x)}{2x^2 + \tan^2(\sqrt{3}x)}$$

Q.27.(b) Let $f(x) = \begin{cases} a + \frac{|x|}{x}, & \text{if } x < 0 \\ x + b + 2, & \text{if } 0 \le x < 1 \\ 2ax, & \text{if } x \ge 1 \end{cases}$ **[6]**

Find the value of a and b , such that $f(x)$ is continuous at every x.

Q. 28(a). Find the derivative of $f(x) = \sin\left[\tan(\cos x)\right]$ **[3]**

Q.28(b). Use Linear approximation function to find the error in the measurement of the surface area of the sphere, if the radius of sphere is measured as 10.5 inch instead of 10 inch, also find the percentage error. **[4]**

Q.28(c). Use second derivative test to discuss concavity and to find points of inflections of the graph of the function

$$f(x) = 3x^4 - 8x^3 + 6x^2 + 1$$ **[6]**

Q. 29(a). Air is being pumped into a spherical balloon at a rate of 64 ft^3/min . At what rate radius is changing when the radius is 4 ft. **[5]**

Q.29(b). Find $\frac{dy}{dx}$ by using implicit differentiation: **[5]**

$$x^2 + x \tan^{-1} y = \sin(xy) .$$

Q.29(c). Find the centre, the foci and the vertices of the ellipse **[5]**

$$x^2 + 4x + 4y^2 - 8y + 4 = 0 .$$