

Linear interpolation example

Today's date is December 5, 2005. A bank needs to determine a Libor rate with a maturity of January 19, 2006, which is approximately 1½ months from today. Rate source is BBA Libor. There is no current Libor quote available for the required maturity, however, so it is necessary to estimate the unknown rate by means of linear interpolation.

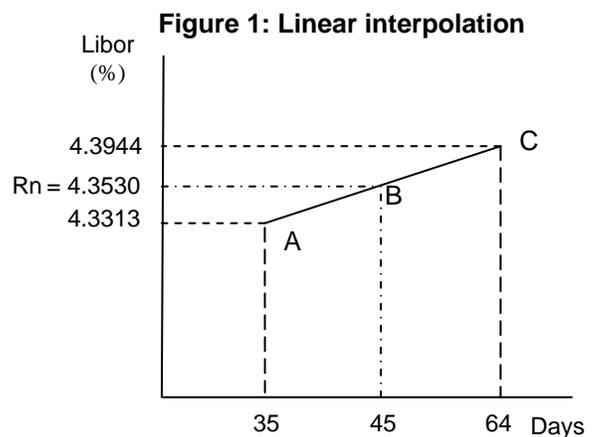
Solution using linear interpolation. Let R_n denote the unknown Libor rate. The closest one can get are the 1-month Libor rate (R_1) and the 2-month rate (R_2), which bracket the unknown rate from above and below. Linear interpolation, which calculates the unknown rate as if it lies on a straight line between the two rates, is the simplest way to calculate the unknown rate. It proceeds as follows.

Today's date:	December 5, 2005
Maturity date of unknown rate:	January 19, 2006
Days to maturity of unknown rate (t_n):	45
Today's 1-month BBA Libor (R_1):	4.3313%
Maturity date of R_1 :	January 9, 2006 ¹
Days to maturity of R_1 (t_1):	35
Today's 2-month Libor (R_2):	4.3944%
Maturity date of R_2 :	February 7, 2006
Days to maturity of R_2 (t_2):	64

Figure 1 shows the relationship between the two rates and days to maturity. Linear interpolation assumes that the unknown rate (R_n) lies on the line (AC) between the two known rates. Because AC is linear, that is, a straight line, the slope of the line (AB) connecting R_1 and R_n is the same as the slope of line AC. Using the "rise over run" formula for the slope of the line, we solve for R_n as follows:

$$\begin{aligned}
 R_n &= R_1 + \frac{R_2 - R_1}{t_2 - t_1} \times (t_n - t_1) \\
 &= 4.3313\% + \frac{4.3944\% - 4.3313\%}{64 - 35} \times (45 - 35) \\
 &= 4.3313\% + 0.00218\% \times (10) = 4.3530\%
 \end{aligned}$$

The interpolated rate is 4.3530%, which lies between the two known rates.



¹ The value date for BBA USD Libor rates is normally two days after the fixing date, so the maturity date for 1M Libor fixed on December 5 would normally be January 7, 2006, however, falls on a Saturday. According to the Modified Following business day convention, which the BBA uses for Libor, the maturity date moves forward to the next "good" business day so long as it is in the same month. If the next good day lies in the following month, however, the maturity date goes back to the last good business day preceding the weekend.

General formula. The following is a derivation of the general version of the above equation. Days to maturity should be adjusted when necessary using the business day convention appropriate to the rate source used. Given two known rates and their maturities:

- R1 = Known rate with shorter maturity
- t1 = Days to maturity of R1
- R2 = Known rate with longer maturity
- t2 = Days to maturity of R2
- Rn = Unknown rate between R1 and R2
- tn = Days to maturity of Rn

find unknown rate Rn with known maturity tn ($t1 < tn < t2$) based on the assumption that the slope of line AB is equal to the slope of line AC as follows.

$$\begin{aligned}\frac{R_n - R_1}{t_n - t_1} &= \frac{R_2 - R_1}{t_2 - t_1} \\ \Rightarrow R_n - R_1 &= \frac{R_2 - R_1}{t_2 - t_1} \times (t_n - t_1) \\ \Rightarrow R_n &= R_1 + \frac{R_2 - R_1}{t_2 - t_1} \times (t_n - t_1)\end{aligned}$$

which is a general formula of linear interpolation.

Another commonly used—and mathematically equivalent—version of the linear interpolation formula is the following:

$$R_n = \frac{[R_1 \times (t_2 - t_n)] + [R_2 \times (t_n - t_1)]}{(t_2 - t_1)}$$

In the preceding example, the solution would work as follows:

$$R_n = \frac{[4.3313\% \times (64 - 45)] + [4.3944\% \times (45 - 35)]}{(64 - 35)} = 4.3530\%$$

which is the same result as that obtained using the other formula.